Designing, Not Checking, for Policy Robustness: An Example with Optimal Taxation

Benjamin B. Lockwood, *University of Pennsylvania Wharton School,* United States of America*, and NBER,* United States of America

Afras Sial, *University of Pennsylvania Wharton School,* United States of America

Matthew Weinzierl, *Harvard Business School,* United States of America*, and NBER,* United States of America

Executive Summary

Economists typically check the robustness of their results by comparing them across plausible ranges of parameter values and model structures. A preferable approach to robustness—for the purposes of policymaking and evaluation—is to design policy that takes these ranges into account. We modify the standard optimal income tax model to include the policymaker's subjective uncertainty over parameter values, and we characterize robust optimal policy as that which maximizes expected social welfare. After calibrating uncertainty over the elasticity of taxable income from past empirical work and novel survey data on economists' beliefs, we compare the implied robust optimal marginal tax rates to the alternative benchmark policy based on the best point estimates of relevant parameters. Our results suggest that robust optimal marginal tax rates are typically more progressive than benchmark analyses, raising top marginal tax rates by 5–7 percentage points and generating modest expected welfare gains.

Policymaking must proceed in the face of widespread uncertainty.[1] Despite our best efforts, economists cannot provide policymakers with definitive estimates of most, if any, of the inputs to optimal policy models (much less guarantee that we are using the right models). Therefore, economists studying optimal policy design have a responsibility to ensure that their results are robust to plausible uncertainty about the model inputs. In this paper, we argue that the current approach to robustness used by most

Tax Policy and the Economy, volume 35, 2021.
 https://doi.org/10.1086/713492

economists is flawed as a guide to policymaking and evaluation, however natural it may be for academic research, and we propose an alternative.

A typical robustness analysis is designed for the use of journal editors and referees, not policymakers. In particular, the results obtained in a baseline case are typically labeled "robust" when they are close—in some unspecified but generally understood sense—to the results obtained at several points within a plausible space of parameter values and range of model specifications. This approach provides reassurance that the results are not special or fragile; that is, that they do not rely on a particular calibration or modeling choice. But this approach is at odds with how economists themselves advise policymakers to confront limits to their understanding, and it is therefore a disservice to policymakers (and their economic advisors). Instead, most economists believe rational policymaking in the face of uncertainty is nothing other than expected social welfare maximization, in which policymakers ought to take into consideration the probabilities and implications of the full range of plausible parameter values when choosing policy.[2] In other words, policies ought to be robust to parameter and model uncertainty by design.

The expected social welfare maximization approach has at least two advantages as a guide to robust policy. First, it makes it more difficult to ignore nonlinearities in "outlier" results from far-off parts of the parameter space. Under the current approach, if some combination of parameter values is unlikely but not impossible, and yet has substantial implications for the results, it may easily escape the analyst's attention and encourage a false sense of security. This risk is especially great in complex situations with a number of uncertain parameters, when relying on the researcher's best judgment may be insufficient. Expected welfare maximization avoids this problem, as it is designed to be sensitive to any extreme welfare implications. Second, expected welfare maximization delivers a rigorously determined optimal policy compromise in the face of uncertainty, whereas the current method provides no guidance on how the results under baseline and "outlier" cases are to be combined. It is striking, upon reflection, how seldom the results of robustness checks under the current approach lead to recommendations for any adjustment to the baseline optimal policy.

To demonstrate how using expected social welfare maximization modifies standard analytical and quantitative optimal policy results, we focus on the much studied optimal income tax problem.[3] Working within the standard Mirrlees (1971) model, we assume that its key parameters may be state dependent and that the tax designer, who maximizes expected social welfare, is uncertain over the true state of the world. The resulting necessary conditions for a robust optimal tax policy, and the policy implied by them under a specific calibration, can be compared with two benchmarks: an "average optimal policy," which equals the probability-weighted average of policies computed for each possible value of these parameters, and the "best estimate optimal policy" based on the expected values of these parameters.

Our new analytical results clarify how the robust optimal tax policy more effectively responds to relationships among uncertain parameters than do the two benchmark policies. For example, in one scenario considered below, we find that the necessary conditions for robust optimal marginal income taxes depend on the inverse of the expected value (across states) of the product of the elasticity of taxable income (ETI) and the density of the income distribution (using the notation below, $1/E_s[\zeta_s(y)h_{s,T}(y)]$) rather than the expected value of the inverse of the product of those parameters ($E_s[1/(\zeta_s(y)h_{s,T}(y))]$, upon which depends the average optimal policy) or the inverse of the product of their expected values ($1/(E_s[\zeta_s(y)]E_s[h_{s,T}(y)])$, upon which depends the best estimate optimal policy).

Our quantitative results indicate that, for plausible levels of uncertainty, the robust optimal tax policy remains broadly similar to the policy chosen under certainty, but instructive differences exist. To calibrate the model, we must construct a subjective probability distribution over key parameter values. For purposes of concreteness, we focus our analysis on a central parameter of optimal income taxation models: the long-term ETI. To calibrate the subjective distribution, we draw on two data sources: the vast existing empirical literature estimating the ETI, and a novel survey of academic economists.[4] Then, for each value of that elasticity, we infer an income-earning ability distribution from current data on income and the tax system. We are thus able to construct state-dependent joint probability distributions for two of the dimensions of uncertainty facing tax policy designers and use them to compare the robust optimal policy to the two benchmark policies. We find that the robust optimal tax schedule retains the now standard U-shape discussed in Diamond (1998) and Saez (2001). At the same time, the robust optimal tax system is generally more progressive and welfare improving. In our baseline case, we calibrate uncertainty using our pilot survey of public finance economists, in conjunction with empirical estimates of the modern US income distribution, and we allow the policymaker's budget constraint to bind only in expectation. We find that robust optimal policy raises marginal and average tax rates on incomes greater than \$100,000 by approximately 7 percentage points and 4 percentage points, respectively, relative to the best estimate policy. Average tax rates are reduced by 2–7 percentage points at lower incomes, and the robust policy generates annual welfare gains of 1.1% of consumption, or $191 billion. When we require the constraint to bind in each state, the robust optimal policy raises marginal and average tax rates at higher incomes by about 5 and 3 percentage points, respectively, and generates more modest annual welfare gains of 0.2% of consumption, or $54 billion.

Finally, we note that our alternative approach in no way undermines, and may improve, the most important benefit of conventional robustness checks. Both scholars and policymakers want to know whether, in what ways, and by how much a policy based on the best of their knowledge will be substantially in error if they turn out to have been wrong. This desire makes it natural to compare two policies: the optimal policy given baseline parameter values (what we call the best estimate optimal policy) and the optimal policy given substantially different values. Under our approach, the first of these two policies is replaced by the optimal robust policy, which maximizes expected social welfare over the possible range of parameter values. Therefore, the results of a traditional robustness analysis can still teach us how costly are deviations of the actual parameter values from our beliefs, but we now measure these costs relative to a rationally constructed robust policy.

I. Analytical Results without and with Uncertainty

To make our approach concrete and demonstrate its practicality, we work within the currently dominant optimal tax framework based on Mirrlees (1971) and developed by Diamond (1998) and Saez (2001), among many others. The Mirrleesian literature has expanded along a number of dimensions, with each expansion introducing greater sophistication and complexity to the already demanding model. But the basic model's core result on marginal tax rates (i.e., distortions to individuals' consumption-leisure trade-offs) relies on a remarkably short list of parameters, one of which is the long-term elasticity of taxable (labor) income.[5] Others include the shape of the income (or income-earning ability) distribution and marginal welfare weights along the income (or ability) distribution.[6]

Uncertainty is pervasive, however, over the values of these parameters upon which Mirrleesian optimal tax results depend. Taxable income elasticities have been exhaustively studied for decades, but their long-term values in response to substantial tax changes remain elusive given the limits of econometrics in a complex and dynamic world.[7] The income distribution is relatively well understood at a point in time, but its long-term evolution and (non-tax-policy) determinants remain only vaguely understood.[8] Even marginal social welfare weights, which for decades were viewed as arising in a straightforward way from a utilitarian social welfare function, have recently come up for debate, with some authors finding support for quite different values.[9]

Our model policymaker takes this uncertainty into account by imagining each possible collection of parameter values as a state of the world, specifying a probability distribution over states of the world, and choosing tax policy to maximize expected social welfare. This probability distribution can be understood as a Bayesian policymaker's prior, which may be subject to change on the basis of new information. In particular, we allow for a range of possible values of the taxable income elasticity and assume functional forms for the income distribution and marginal social welfare weights. Then each possible elasticity implies—given observed pretax and posttax income distributions and a tax system—an underlying income-earning ability distribution and a set of marginal social welfare weights, together generating a collection of state-specific parameter values for the model policymaker to use.

A. Linear Taxation

To build intuition, we begin with an optimal linear tax model of the kind considered in Sheshinski (1972). The tax system is defined by a lump-sum grant b and constant marginal rate t.

Certainty Benchmark

Individuals differ in their unobservable income-earning ability type, indexed by $i = \{1, 2, \ldots, I\}$, and choose labor effort to maximize utility $U(c^i, y^i)$ where c^i is individual i's after-tax income (i.e., consumption) and y^i is pretax income (i.e., the product of ability and effort). We use ζ^i to denote the compensated elasticity of the individual's taxable income with respect to $1 - t$. For the purposes of this section, we assume utility is quasi-linear in consumption so that there are no income effects of taxation on an individual's optimization, and thus ζ^i is also the uncompensated ETI. An individual earning income y^i pays tax $T(y^i) = -b + ty^i$, so the individual's budget constraint is $c^i = (1 - t)y^i + b$, and individual i's indirect utility can be denoted $V^i(1 - t, b)$. We denote average income with $\bar{y} = \Sigma_i p^i y^i$, where p^i is the fixed population proportion of type i in the economy.

The tax authority's objective is to maximize a simple-sum utilitarian measure of social welfare: $\Sigma_i p^i V^i(1 - t, b)$. This form for the objective is standard—though not uncontroversial—in the literature. We define

$$g^i = \frac{\left(\frac{dV^i}{db}\right)}{\lambda}, \tag{1}$$

to denote the marginal social welfare of income for individual i, equal to individual i's marginal utility of consumption (the numerator) normalized by the marginal value of public funds λ.[10]

The tax authority's budget constraint is

$$t\sum_i p^i y^i = b + R, \tag{2}$$

where R is exogenous required public goods spending.

We use the perturbation method made familiar (in the nonlinear tax context) by Saez (2001) to derive the necessary conditions for optimal policy. The strategy is to note that at the optimum, social welfare is maximized with respect to the tax rate t, and so a small change dt must have no first-order impact on welfare when all its component parts are summed.[11]

The effects of the perturbation dt can be collected into two classes: mechanical and behavioral. The mechanical effect includes a revenue increase from higher taxes on each agent's income, equal to a revenue change of $y^i dt$ from each individual i, and a consequent mechanical reduction in welfare of $g^i y^i dt$ for each individual i. The total mechanical effect, dM, is therefore

$$dM = \sum_i p^i (1 - g^i) y^i dt. \tag{3}$$

The behavioral effect arises from each individual i adjusting earnings by $-(dy^i / d(1-t))dt$, which thereby changes tax revenue. The total behavioral effect, dB, is

$$dB = -t\sum_i p^i \frac{dy^i}{d(1-t)} dt. \tag{4}$$

If the initial policy is optimal, then $dM + dB = 0$. With some simplification (and recalling that $(dy^i / d(1-t)) \cdot ((1-t)/y^i) = \zeta^i$), the optimal linear tax rate t satisfies

$$\frac{t}{1-t} = \frac{E[y^i - g^i y^i]}{E[\zeta^i y^i]} = \frac{-\text{Cov}[g^i, y^i]}{E[\zeta^i y^i]}. \tag{5}$$

We additionally assume $\zeta^i \equiv \zeta$ is constant across individuals, and we retain this assumption throughout the rest of this section. Then equation (5) can be expressed more simply as

$$\frac{t}{1-t} = \frac{-\mathrm{Cov}\left[g^i, \frac{y^i}{\bar{y}}\right]}{\zeta}. \tag{6}$$

In words, expression (6) says that the optimal linear tax rate t is positive if the value society places on an additional unit of income is lower for higher-income individuals ($\mathrm{Cov}[g^i, (y^i/\bar{y})] < 0$), and that tax rate is larger the more that value declines with income. However, the optimal linear tax rate is smaller the greater the elasticity (and thus efficiency costs) of taxation (i.e., the larger is ζ). Equations (5) and (6) closely resemble the many-person Ramsey tax rule for the optimal redistributive linear commodity tax, as derived in Diamond (1975).

This paper is motivated by the observation that the result in equation (6) is obtained under the assumption that its terms are known with certainty by the tax authority. In reality, the parameters on the right-hand side of this expression are estimable only with great uncertainty, if at all. We turn now to understanding how robust optimal linear taxation manages that uncertainty.

Adding Uncertainty: The Fully Prespecified Robust Optimum

Now, we consider a setting in which multiple possible states of the world, indexed by $s = \{1, 2, \ldots, S\}$, arise with perceived probabilities π_s, where $\Sigma_s \pi_s = 1$.[12] Each state corresponds to different realized values of the components of expression (6) over which the tax authority is uncertain when designing policy. Individuals' indirect utility from a given tax policy may now differ across states, denoted $V_s^i(1-t, b)$, and as a result, marginal welfare weights are also now state specific:

$$g_s^i = \frac{\left(\frac{dV_s^i}{db}\right)}{\lambda}.$$

Note that at the optimum, the policymaker is indifferent between marginal spending on public funds and a marginal increase in the prespecified lump-sum grant, implying $\lambda = \Sigma_s \pi_s \Sigma_i p_s^i (dV_s^i/db)$. Incomes, population shares, and elasticities of taxable income are also state specific, denoted y_s^i, p_s^i, and ζ_s^i. We denote the within-state average of any variable x_s^i as $\bar{x}_s = \Sigma_i p_s^i x_s^i$ and the across-state average as $\bar{x} = \Sigma_s \pi_s \bar{x}_s$.

The tax authority must specify one tax system $\{b^*, t^*\}$ that will apply regardless of which state obtains; asterisks are used to denote the robust optimal tax policy. Conceptually, we have in mind choosing a tax policy that is expected to apply, unchanged, over a long time horizon, so we abstract from concerns about transition dynamics. The tax authority's objective is to maximize expected social welfare as before, but now this maximization involves an expectation over states:

$$\sum_s \pi_s \sum_i p_s^i V_s^i(1 - t, b). \tag{7}$$

In our baseline derivation, we constrain the tax authority to a fully prespecified, state-invariant policy, with a fixed linear rate t and lump-sum grant $b(t) = t\Sigma_s \pi_s \Sigma_i p_s^i y_s^i - R$. Note that framing the problem in this way implies that the budget constraint must bind only in expectation; it effectively allows for transfers of resources across states s.[13] A very plausible alternative assumption is to require that the budget constraint must not be violated within each state. This case necessarily requires that some parameter of the tax—most naturally, the budget constraint b—is state contingent; we consider this possibility in the next section.

Before characterizing robust optimal policy, we note how our approach differs from some related lines of research.

In a series of contributions dating back 3 decades, Charles Manski studied the problem of "microeconomic" policymaking in the presence of uncertainty. He mentions the potential application to settings of optimal taxation explicitly in Manski (2009, 146) with the following remarkable paragraph, which we quote in full:

> Research on optimal income taxation illustrates the problem. Stimulated by Mirrlees [1971], many theoretical studies have derived optimal tax schedules under the assumption that the planner knows how the tax schedule affects labor supply. However, our knowledge of the actual responsiveness of labor supply to income taxes remains limited, despite the strenuous effort of empirical economists to shed light on the matter. For this reason, among others, research on optimal income taxation has not played much of a role in practical analysis of tax policy.

This motivation is tightly related to our own. Despite this similarity, however, Manski proposes a rather different method for policy selection. He notes that although one might act as a "Bayesian planner" and choose policies that maximize expected welfare with respect to a subjective probability distribution over policy states, in some settings, the formation of that subjective distribution may not be straightforward. Instead, he proposes a "minimax regret" procedure in which the planner seeks to ensure that welfare losses relative to the (ex post) optimal policy are not too large. We view our work as complementary, exploring the optimal policy as a function of the subjective probability distribution across states, in the event that such a distribution can be credibly formed.

Separately, Hansen and Sargent (2001) introduce to macroeconomic theory the techniques of robust optimal control in which the primary concern is to avoid worst-case scenarios (see Williams 2008). Formally, Hansen and Sargent's policymakers proceed as if they are playing a game against a malevolent Nature that selects the worst realization of parameter or model uncertainty given the chosen policy. As a result, the robust policy literature within macroeconomics typically emphasizes maximin policy objectives. For concreteness and simplicity, we assume all concavity arises through the individual's utility functions, and the policymaker is risk neutral in aggregate welfare across states. Our expected social welfare approach therefore reacts less strongly to worst-case scenarios; although such scenarios are naturally weighted more heavily due to the fact that marginal utility of consumption is higher in bad states, the policymaker does not place additional weight (let alone infinite weight) on those outcomes.[14]

More similar to our approach is Kasy (2018), which advocates using machine learning techniques to maximize expected welfare when parameters are uncertain with a Gaussian process prior distribution. That paper applies its model to a simulation with health insurance copays and, consistent with our results, finds that accounting for uncertainty has a substantial impact on optimal policy.

Returning to our analysis, applying the same perturbation method as above, we obtain parallel expected mechanical and behavioral effects, which we denote dM^e and dB^e. The expected mechanical effect, analogous to equation (3), is

$$dM^e = \sum_s \pi_s \left[\sum_i p_s^i (1 - g_s^i) y_s^i dt \right]. \tag{8}$$

The expected behavioral effect, analogous to equation (4), is

$$dB^e = -t \sum_s \pi_s \left[\sum_i p_s^i \frac{dy_s^i}{d(1-t)} dt \right]. \tag{9}$$

In this scenario with uncertainty, a necessary condition for optimality is that the tax authority does not view such a perturbation as yielding a net welfare gain in expectation. Setting $dM^e + dB^e = 0$, and employing the assumption that ζ_s^i is homogeneous within states (denoted ζ_s), the optimal t^* is characterized by the following proposition.

Proposition 1. The optimal fully prespecified robust linear income tax rate satisfies

$$\frac{t^*}{1-t^*} = \frac{-\mathrm{Cov}\left[g_s^i, \frac{y_s^i}{\bar{y}}\right]}{\bar{\zeta}\left(1 + \mathrm{Cov}\left[\frac{\zeta_s}{\bar{\zeta}}, \frac{\bar{y}_s}{\bar{y}}\right]\right)}. \tag{10}$$

Proof. Setting $dM^e + dB^e = 0$ from equations (8) and (9), and rearranging, yields

$$\frac{t^*}{1-t^*} = \frac{\sum_s \pi_s \bar{y}_s - \sum_s \pi_s \left[\sum_i p_s^i g_s^i y_s^i\right]}{\sum_s \pi_s [\zeta_s \bar{y}_s]}. \tag{11}$$

We can manipulate the second summation in the numerator as $\Sigma_s \pi_s[\Sigma_i p_s^i g_s^i y_s^i] = E_{i,s}[g_s^i y_s^i] = E_{i,s}[g_s^i]E_{i,s}[y_s^i] + \mathrm{Cov}[g_s^i, y_s^i]$. Then using the fact that we normalize $E[g_s^i] = 1$, the numerator of equation (11) reduces to $-\mathrm{Cov}[g_s^i, y_s^i]$. Similarly, the denominator can be decomposed as $\Sigma_s \pi_s[\zeta_s \bar{y}_s] = E_s[\zeta_s \bar{y}_s] = E_s[\zeta_s]E_s[\bar{y}_s] + \mathrm{Cov}[\zeta_s, \bar{y}_s]$. Factoring out $\bar{\zeta}$ and dividing the numerator and denominator by $\bar{y}$ results in expression (10). QED

To understand the adjustments the tax authority makes to achieve robustness in the face of uncertainty, it is useful to compare the result in proposition 1 with the expression for the optimal linear tax rate without uncertainty in equation (6). The expressions are similar, with the exception of the term $1 + \mathrm{Cov}[(\zeta_s/\bar{\zeta}), (\bar{y}_s/\bar{y})]$ in the denominator of equation (10). This term points to a key force that determines robustly optimal policies: if elasticities are low in states of the world with high incomes—meaning that $\mathrm{Cov}[(\zeta_s/\bar{\zeta}), (\bar{y}_s/\bar{y})] < 0$—then taxes are generally less distortionary, implying that the optimal tax rate is higher than in a setting with certainty.

To formalize the effect of robust policy, we compare the result in proposition 1 to two natural alternatives. The first is the "best estimate optimal linear tax rate," that is, the optimal linear tax computed as if the policymaker's best point estimates (across states) of each parameter value were known with certainty. To express this, we write the across-state, within-person average of a parameter x_s^i as $\bar{x}^i = \Sigma_s \pi_s x_s^i$. Then this policy can be found by substituting the best estimates for each parameter into the expression for the optimal linear tax rate with certainty:

$$\frac{t}{1-t} = \frac{-\mathrm{Cov}\left[\bar{g}^i, E_s\left[\frac{y_s^i}{\bar{y}_s}\right]\right]}{\bar{\zeta}}. \tag{12}$$

Alternatively, the tax authority might solve for the optimal linear tax rate given each possible state of the world (using a state-specific version of eq. [6]) and then take the expectation of those values across states. We denote the resulting policy with $\{\bar{b}, \bar{t}\}$ and refer to it as the "average optimal linear tax rate," which satisfies

$$\frac{\bar{t}}{1-\bar{t}} = E_s\left[\frac{-\mathrm{Cov}\left[g_s^i\,\frac{y_s^i}{\bar{y}_s}\right]E_s[\bar{y}_s]}{\zeta_s E_s[\bar{y}_s]}\right]. \tag{13}$$

Comparing results in equations (10), (13), and (12) reveals the main difference between robust optimal linear taxes and the two alternatives.

Mathematically, understanding this difference starts with the observation that expressions (10), (13), and (12) are of nearly the same structure. That is, all three rates (t^*, $\bar{t}$, and t) depend in the same directions on the same multiplicative terms: the product of the covariance (between welfare weights and relative income) and mean income in the numerator and the product of the ETI and mean income in the denominator. How they differ is in the level at which the expectation over states is taken: at the level of these multiplicative terms for the robust optimal rate t^*, at the level of the ratio of these terms for the average optimal rate $\bar{t}$, and at the level of each factor within these terms for the best estimate optimal rate t. As a result, neither the average optimal linear tax rate in equation (13) nor the best estimate linear tax rate in equation (12) utilizes information most effectively to maximize expected welfare.

Economically, the intuition behind the difference is that the robust tax policy is sensitive—to the optimal degree—to complementarities in the effects of uncertainty's resolution across states. For example, suppose there are three states $s = \{1, 2, 3\}$, and their elasticities are symmetrically distributed, such as $\zeta_1 = 0.25$, $\zeta_2 = 0.50$, and $\zeta_3 = 0.75$. Suppose further that the high-elasticity state also has a substantially greater average income than in the other states, such that $E_s[\zeta_s\bar{y}_s] > E_s[\zeta_s]E_s[\bar{y}_s]$. In that case, the robust optimal rate reflects that the distortionary costs of taxation are greater in expectation than what the best estimate optimal rate would suggest because the state in which the ETI is high is also that in which incomes are greater.[15] The conceptual contrast between the robust optimal rate and the average optimal rate is more subtle: although the former takes these complementarities into account separately in the numerator and denominator, the latter takes them into account through (i.e., by taking the expectation over states of) their ratio. If the values of the multiplicative terms in the numerator and denominator are correlated across states, either positively or negatively, the average optimal linear tax rate will respond not to the ratio of their expected values but to the expected value of their ratio, missing opportunities for more sophisticated robustness to uncertainty.

More colloquially, the robust optimal tax policy tries to protect against worst-case scenarios, take advantage of best-case scenarios, and manage the cases in between rather than just do best in the expected case or choose a simple average of its best state-specific values.

The Optimum When Budgets Must Balance within Each State

The fully prespecified tax policy above is perhaps extreme in that it implies that resources can be transferred across states. In many settings, this may be unrealistic; real productive capacity might vary across states, for example, with no counterparty available to provide across-state insurance. Therefore, we also consider a natural alternative assumption: that the budget constraint binds within each state. However, this assumption introduces a complication. On the one hand, it implies that some feature of the tax schedule must be state contingent, and the nature of that state contingency must be prespecified. On the other hand, it does not seem realistic that the entire tax system can be made state contingent. (If it could, then the problem of robustness dissolves, as the fully optimal tax policy is to simply select the optimal tax policy as a function of the parameters that arise in each state.)

We choose a middle ground: we assume that the tax rate t must be prespecified (or, in the analysis of nonlinear tax schedules to follow, the full schedule of marginal tax rates), whereas the lump-sum grant b is allowed to adjust in each state to achieve budget balance: $b_s(t) = t\Sigma_i p_s^i y_s^i - R$.[16] This setting constrains the tax authority more severely than our previous setting, as it disallows transfers of resources across states. Which setting is preferable depends on, among other factors, how completely this model captures the total fiscal policy problem of the tax authority. If it is viewed as a narrow portion (e.g., the part of the policymaking problem devoted to redistribution but separate from issues of trade, regulation), then the tax authority may be described as choosing a fully prespecified tax policy that is self-funding in expectation with imbalances absorbed elsewhere. However, if this model is viewed as an abstraction for the macroeconomy as a whole, then the second setting may be preferable because it more accurately reflects the necessary adjustment to the lump-sum transfer that the tax authority will have to make once the true state obtains.

In this setting, the tax authority must take into account that the chosen tax rate will have implications for the lump-sum grant that vary, perhaps dramatically, by state. An implication of this modification is that the marginal value of public funds may differ across states, since resources cannot be reallocated to equate them. Because budget constraints now apply within each state, it is useful to define the marginal value of public funds within each state:

$$\lambda_s = \sum_i p_s^i \left(\frac{dV_s^i}{db} \right). \tag{14}$$

This modification alters the expected mechanical effect of a small tax change, since the marginal funds generated in state s are worth λ_s/λ in expected public funds. Therefore, the expected mechanical effect is now

$$dM^e = \sum_s \pi_s \left[\sum_i p_s^i \left(\frac{\lambda_s}{\lambda} - g_s^i \right) y_s^i dt \right]. \tag{15}$$

Likewise, the behavioral effect is also modified:

$$dB^e = -t \sum_s \pi_s \frac{\lambda_s}{\lambda} \left[\sum_i p_s^i \frac{dy_s^i}{d(1-t)} dt \right]. \tag{16}$$

Employing the same perturbation method as above, the robust optimal policy under within-state budget balance is given by the following proposition:

Proposition 2. The optimal robust linear income tax rate when the budget constraint binds in each state satisfies

$$\frac{t^*}{1-t^*} = \frac{-\mathrm{Cov}\left[g_s^i, \frac{y_s^i}{\bar{y}} \right] + \mathrm{Cov}\left[\frac{\lambda_s}{\lambda}, \frac{y_s^i}{\bar{y}} \right]}{\bar{\zeta}\left(1 + \mathrm{Cov}\left[\frac{\zeta_s}{\bar{\zeta}}, \frac{\bar{y}_s}{\bar{y}} \right] + \mathrm{Cov}\left[\frac{\lambda_s}{\lambda}, \frac{\zeta_s \bar{y}_s}{\bar{\zeta}\bar{y}} \right] \right)}. \tag{17}$$

Proof. Setting $dM^e + dB^e = 0$ from equations (15) and (16), and rearranging, yields

$$\frac{t^*}{1-t^*} = \frac{\sum_s \pi_s \lambda_s \bar{y}_s - \lambda \sum_s \pi_s \left[\sum_i p_s^i g_s^i y_s^i \right]}{\sum_s \pi_s [\lambda_s \zeta_s \bar{y}_s]}. \tag{18}$$

We then decompose the expectations using the definition of covariance to yield the expression in the proposition. QED

Compared with equation (10) above, equation (17) incorporates the state-specific marginal values of public funds in both the numerator and the denominator. These additions capture two forces: on the one hand, if increasing the tax rate generates extra revenue in states where marginal utilities are high on average (large λ_s) then the tax should be higher; on the other hand, if increasing the tax rate depresses income in those states especially strongly, it should be lower. These forces are absent in the setting where resources can be transferred across states.

B. Nonlinear Optimal Taxation

We now remove the linearity restriction on the tax system to characterize robust optimal nonlinear income taxes, thus connecting more closely to the main Mirrleesian literature.

Certainty Benchmark

Individuals are now assumed to belong to a continuum of (possibly multidimensional) types. To reflect this difference from the preceding section, we denote the continuous index of types by $\theta \in \Theta$, which is distributed according to $F(\theta)$ with associated density $f(\theta)$. Individuals choose, taking the tax system as given, a level of labor effort to maximize individual utility. The product of labor effort and type is income y, which is distributed (conditional on the tax function T) according to $H_T(y)$ with associated density $h_T(y)$. Let $\zeta(y)$ denote the compensated labor supply elasticity, and we now generalize the theoretical model to allow for income effects as well, denoted $\eta(y)$. The nonlinear income tax $T(y)$ is chosen by the tax authority to maximize social welfare subject to the feasibility constraint $\int_{y>0} T(z) h_{s,T}(z) dz \geq R$, where R is exogenous public goods expenditure.

As with the linear tax case, we use the perturbation method to derive our results. Consider a small increase in the marginal tax rate $d\tau$ in an interval of size ε around some earnings level y^*. The mechanical effect of this increase has three components. First, it raises revenues equal to $\varepsilon d\tau \int_{y^*}^{\infty} h_T(z) dz$ from those earning y^* and more. Second, it directly reduces after-tax income among these individuals, with an effect on social welfare of $\varepsilon d\tau \int_{y^*}^{\infty} (-g(z)) h_T(z) dz$, where $g(y)$ is (as before) the marginal social welfare weight on y-earners in terms of public funds. Third, this after-tax income reduction for individuals with earnings above y^* may cause them to raise their earnings through an income effect, generating an increase in revenue for the tax authority. Denote this fiscal externality as $\varepsilon d\tau \int_{y^*}^{\infty} (-T'(z)/(1 - T'(z))) \eta(z) h_T(z) dz$. For notational simplicity, let $\hat{g}(y) = g(y) - (T'(y)/(1 - T'(y)))\eta(y)$ denote the combination of the latter two effects.[17] We can write the overall mechanical effect, then, as

$$dM = \varepsilon d\tau \int_{y^*}^{\infty} (1 - \hat{g}(z))h_T(z)dz. \tag{19}$$

This tax increase also generates a behavioral effect, as y^*-earners reduce their effort and, thereby, generate a negative fiscal externality equal to

$$dB = \varepsilon d\tau \frac{-T'}{1-T'} \zeta(y^*)y^* h_T(y^*). \tag{20}$$

At the optimum, the tax system is optimal only if the sum of these effects is zero. That is, if $T(y)$ is optimal, then the following condition on the marginal tax rate $T'(y)$ holds:

$$\frac{T'(y)}{1-T'(y)} = \frac{1}{y\zeta(y)h_T(y)} \int_{y}^{\infty} (1 - \hat{g}(z))h_T(z)dz. \tag{21}$$

Adding Uncertainty: The Optimal Fully Prespecified Robust Nonlinear Tax

As in the linear setting, we now allow for uncertainty over the parameters that matter for optimal taxation. In particular, we imagine that there are multiple possible states of the world indexed by s, each arising with probability π_s such that $\Sigma_s \pi_s = 1$. We denote uncertainty through state dependence of the individual-type distributions $F_s(\theta)$ and densities $f_s(\theta)$, endogenous income distributions $H_{s,T}(y)$ and densities $h_{s,T}(y)$, compensated elasticities of taxable income and income effects $\zeta_s(y)$ and $\eta_s(y)$, and marginal welfare weights $g_s(y)$. We assume that the tax authority must prespecify the entire tax function—that is, both the nonlinear income tax schedule $T(y)$ and the lump-sum grant $-T(0)$—to apply in all states so that the tax authority's budget constraint is satisfied in expectation: $\Sigma_s \pi_s [\int_{y>0} T(z)h_{s,T}(z)dz] \geq R$.

The perturbation method proceeds as before. Consider a small increase in the marginal tax rate $d\tau$ in an interval of size ε around some earnings level y^*. The expected mechanical effect is expression (19) modified for multiple states:

$$dM^e = \varepsilon d\tau \sum_s \pi_s \left[\int_{y^*}^{\infty} (1 - \hat{g}_s(z))\, h_{s,T}(z)dz \right]. \tag{22}$$

The expected behavioral effect is a similar modification of expression (20):

$$dB^e = \varepsilon d\tau \sum_s \pi_s \left[\frac{-T'(y^*)}{1 - T'(y^*)} \zeta_s(y^*) y^* h_{s,T}(y^*) \right]. \tag{23}$$

The sum of these expected effects must be zero under the robust optimal tax, so

$$\frac{T^{*\prime}(y)}{1 - T^{*\prime}(y)} = \frac{E_s\left[\int_y^\infty (1 - \hat{g}_s(z))\, h_{s,T}(z) dz \right]}{y E_s[\zeta_s(y) h_{s,T}(y)]}. \tag{24}$$

Again, it is useful to compare this result to the best estimate optimal tax policy,

$$\frac{T'(y)}{1 - T'(y)} = \frac{\int_y^\infty E_s[(1 - \hat{g}_s(z))] E_s[h_{s,T}(z)] dz}{y E_s[\zeta_s(y)] E_s[h_{s,T}(y)]}, \tag{25}$$

and the average optimal tax policy,

$$\frac{\bar{T}'(y)}{1 - \bar{T}'(y)} = E_s \left[\frac{\int_y^\infty (1 - \hat{g}_s(z)) h_{s,T}(z) dz}{y \zeta_s(y) h_{s,T}(y)} \right]. \tag{26}$$

As with the results in equations (10), (12), and (13) in the linear case, the most noticeable difference among equations (24), (25), and (26) is the level over which the expectation across states is taken. The robust optimal policy at income y takes into account the multiplicative interactions within states between the compensated ETI and the density of the income distribution at y as well as those between the marginal welfare weight at y and that density. In contrast, the best estimate nonlinear income tax takes expectations of the factors within those multiplicative terms, whereas the average optimal nonlinear tax policy turns on the ratio of those multiplicative interactions within states. In other words, the robust optimal policy appropriately reacts to how inputs to the optimal policy function may build upon or offset each other in the various states of the world—that is, how uncertainty across inputs to the model may exacerbate or mitigate the optimal policy design problem.

The Optimal Robust Nonlinear Tax When Budgets Must Balance within Each State

As before, we now also consider a scenario in which the tax authority must balance its budget within each state. Specifically, we require the tax authority to prespecify the nonlinear income tax schedule $T(y)$ that will apply across states, but we adjust the state-specific lump-sum grant to ensure state-by-state budget balance. Formally, the expression for the state-specific lump-sum grant is $b_s = \int_{y>0} T(z) h_{s,T}(z) dz - R$. This setting constrains the tax authority more severely than our previous setting, as it disallows transfers of resources across states.

Using the same perturbation approach, we obtain expressions for the mechanical effect

$$dM^e = \varepsilon d\tau \sum_s \pi_s \left[\int_{y^*}^{\infty} \left(\frac{\lambda_s}{\lambda} - \hat{g}_s(z) \right) h_{s,T}(z) dz \right], \tag{27}$$

and the behavioral effect

$$dB^e = \varepsilon d\tau \sum_s \pi_s \frac{\lambda_s}{\lambda} \left[\frac{-T'(y^*)}{1 - T'(y^*)} \zeta_s(y^*) y^* h_{s,T}(y^*) \right]. \tag{28}$$

In both expressions, the term λ_s denotes the state-specific marginal value of public funds, which measures the welfare impact of a small increase in the state-specific lump-sum grant.

At the optimum, the following condition must hold:

$$\frac{T^{*\prime}(y)}{1 - T^{*\prime}(y)} = \frac{E_s \left[\int_{y}^{\infty} (\lambda_s - \hat{g}_s(z)\lambda) \, h_{s,T}(z) dz \right]}{y E_s [\lambda_s \zeta_s(y) h_{s,T}(y)]}. \tag{29}$$

Expression (29) captures factors quite similar to those in expression (17): tax rates should be higher if they generate more revenue when the state-specific marginal value of public funds λ_s is high (explaining its presence in the numerator), but tax rates should be lower if that marginal value of public funds is high precisely when the ETI is high (explaining the presence of λ_s in the denominator). For our simulations in Section III, we consider both the case in which the tax policy is fully prespecified and this case in which the lump-sum grant (or, more generally, the total level of government spending) adjusts in each state to achieve balance.

II. Evidence on the Extent of Parameter Uncertainty

In this section, we attempt to quantify some of the uncertainty to which the robust optimal tax conditions of the previous section respond. We restrict our attention to the dimensions of uncertainty over which we have relatively good information, that is, over the parameters for which uncertainty is not as great. In particular, we start by focusing on what economists know—and do not know—about the policy-relevant value(s) of the ETI. Then we use recent data on the tax system and income distribution to infer, for each possible value of that elasticity, an underlying income-earning ability distribution. These exercises produce a joint probability distribution for the ETI and the income distribution, allowing us to simulate and compare robust and benchmark optimal policies in the next section.

We note, however, that we are including only a small portion of the risk, uncertainty, and ignorance faced by a tax designer using the optimal policy model above.[18] In particular, we abstract from the many nontax factors that will affect the evolution of the income distribution, and we assume certainty in normative judgments of optimal policy (i.e., in the pattern of welfare weights). In reality, these other dimensions of limits to our understanding—and how they interact with each other and the dimensions we do include—are likely to have additional substantial quantitative implications for robust policy.

A. Elasticity of Taxable Income

To characterize beliefs on the long-term ETI, we turn to two sources of evidence: the existing empirical literature and a new survey of academic economists.

Existing Literature on the ETI

A leading survey of the literature on the ETI (Saez, Slemrod, and Giertz 2012, 43) summarizes the state of knowledge about this parameter's value over the horizon of relevance to optimal tax theory: "Estimates of the elasticity of taxable income in the long run (i.e., exceeding a few years) are plagued by extremely difficult issues of identification, so difficult that we believe that there are no convincing estimates of the long-run elasticity of reported taxable income to changes in the marginal tax rate."[19] Giertz (2010, 409) reaches a similarly discouraging conclusion:

> In most ETI studies, it is more accurate to refer to estimated elasticities as either "short term" or "longer term" (as opposed to "long term"). Long-term responses may be the most important but could take many years before responses are fully observed. These types of changes (which may include some human capital and occupation decisions as well as more traditional investment) are currently beyond the scope of the ETI literature. For that reason, this article eschews the label "long

term" in favor of "longer term" in order to distinguish these estimates from short-term elasticities while also recognizing that they are not truly long term.

A similar point has been forcefully made in a review of the gap between micro and macro estimates of labor supply elasticities by Keane and Rogerson (2012).

Lacking definitive evidence, what do economists believe is a reasonable value for this elasticity? Three examples serve to illustrate the range of beliefs. Diamond and Saez (2011, 173) are skeptical of substantial real responses to marginal income tax rates, concluding that "the elasticity $e = 0.57$ is a conservative upper bound estimate" and that a value of 0.9 would be "extremely high" (172).[20] Giertz (2010, 406) estimates values "from 0.78 to 1.46 over the longer term" of 3–6 years, and Giertz (2009) considers values ranging from 0.2 to 1.0 for his calculations of the efficiency consequences of tax reforms. Jones (2019) incorporates idea generation into the model, and although he acknowledges substantial uncertainty, he prefers parameterizations that imply a responsiveness of aggregate income corresponding to elasticities as high as 2.36.[21] More generally, the difficulty—even impossibility—of taking into account truly long-term factors, such as changes to social norms around work in response to taxation, strongly suggests to us that economists have very limited knowledge of the policy-relevant value for this key parameter.

Novel Survey Evidence on the ETI

We also take a direct route to gauging economists' beliefs about the value of the ETI: we ask them. We designed a novel survey to elicit beliefs about this parameter, in which we ask the following question.

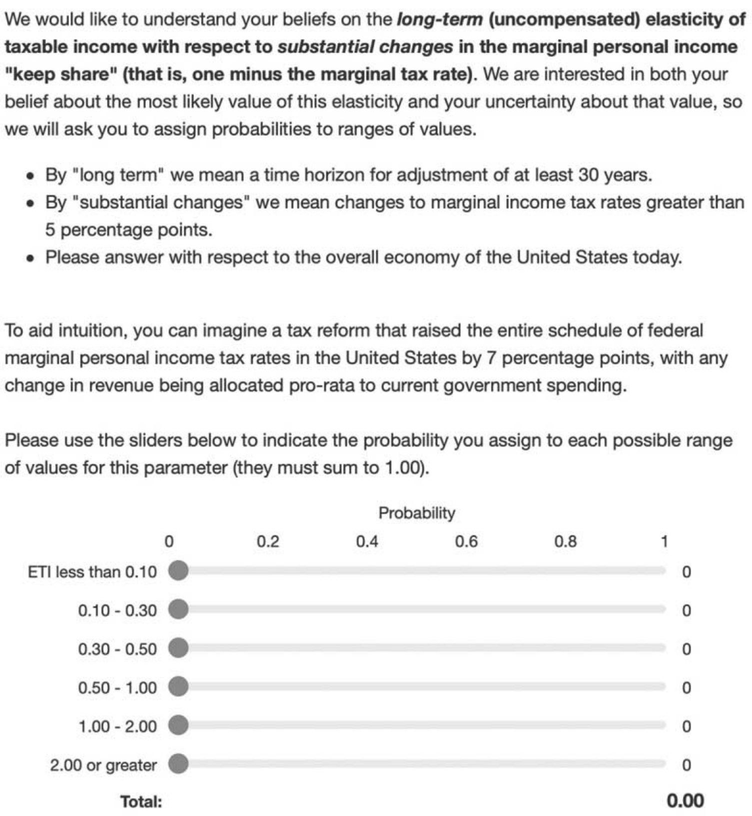
We would like to understand your beliefs on the ***long-term*** **(uncompensated) elasticity of taxable income with respect to *substantial* changes in the marginal personal income "keep share" (that is, one minus the marginal tax rate)**. We are interested in both your belief about the most likely value of this elasticity and your uncertainty about that value, so we will ask you to assign probabilities to ranges of values.

- By "long term" we mean a time horizon for adjustment of at least 30 years.
- By "substantial changes" we mean changes to marginal income tax rates greater than 5 percentage points.
- Please answer with respect to the overall economy of the United States today.

To aid intuition, you can imagine a tax reform that raised the entire schedule of federal marginal personal income tax rates in the United States by 7 percentage points, with any change in revenue being allocated pro-rata to current government spending. Please use the sliders below to indicate the probability you assign to each possible range of values for this parameter (they must sum to 1.00).

We then provide six ranges into which the ETI might fall, and we ask respondents to assign a probability to each range, with final probabilities required to sum to 1. The full survey instrument is shown in figure 1.

We note two features of this question. First, we describe in some detail the parameter about which we are asking. Though perhaps tedious to the respondent, this detail is essential for eliciting the right beliefs for our purposes. In particular, we emphasize that we are interested in the long-run elasticity so that respondents will take into account some of the factors—such as human capital accumulation and social norms adjustment—that short-term elasticity estimates may safely neglect. Second, we elicit a full


We would like to understand your beliefs on the ***long-term*** **(uncompensated) elasticity of taxable income with respect to *substantial changes* in the marginal personal income "keep share" (that is, one minus the marginal tax rate).** We are interested in both your belief about the most likely value of this elasticity and your uncertainty about that value, so we will ask you to assign probabilities to ranges of values.

- By "long term" we mean a time horizon for adjustment of at least 30 years.
- By "substantial changes" we mean changes to marginal income tax rates greater than 5 percentage points.
- Please answer with respect to the overall economy of the United States today.

To aid intuition, you can imagine a tax reform that raised the entire schedule of federal marginal personal income tax rates in the United States by 7 percentage points, with any change in revenue being allocated pro-rata to current government spending.

Please use the sliders below to indicate the probability you assign to each possible range of values for this parameter (they must sum to 1.00).

Fig. 1. Pilot survey question. A color version of this figure is available online.

(subjective) distribution over these value ranges rather than a specific point estimate. This is crucial for exploring the implications of uncertainty for optimal policy, as we will then be able to explore both the optimal policy that is robust to uncertainty across economists and also the robust policy that individual economists should favor in view of their own subjective uncertainty.

We consider the range 0.30–0.50 to be the "central" range in our simulations, as it contains the preferred point estimate from a number of recent prominent surveys (e.g., Chetty et al. 2011; Diamond and Saez 2011; Chetty 2012).

This paper reports results based on a small pilot survey with responses from nine prominent public finance economists. The results of this pilot survey are shown in figure 2, where we plot the mean answer for each point in the discretized distribution.

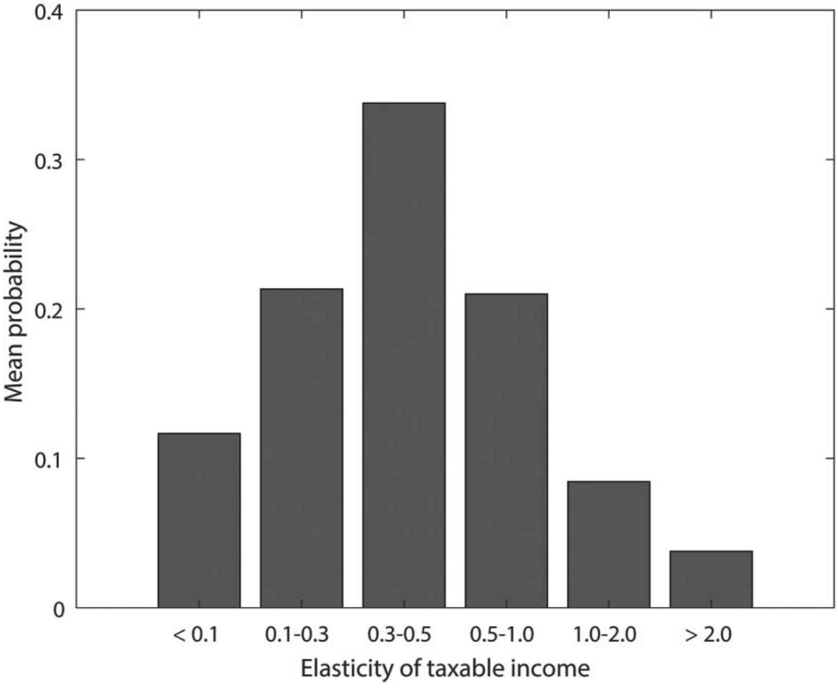


Fig. 2. Probability distribution of the elasticity of taxable income (pilot survey data). This figure plots the mean probability assigned to each range of values of the long-term uncompensated elasticity of taxable income for which we elicited beliefs in our survey of academic economists. Data are based on a pilot survey of nine public finance economists. A color version of this figure is available online.

Combining Evidence on the ETI

The evidence on ζ shows that robust optimal tax policy design confronts both risk and uncertainty. In particular, a robust tax policy designer faces not only the risk posed by the distribution over the possible values for ζ that any individual researcher will suggest but also the uncertainty posed by disagreement over that distribution across researchers. In principle, robust optimal policy might respond differently to these two types of limits to understanding.

We simplify matters and treat risk and uncertainty in the same formal manner: that is, both are managed through expected social welfare maximization. Different possible probability distributions are simply weighted (perhaps uniformly, though differential credence could generate a motive for variation) and summed, yielding a single probability distribution used as the basis for robust policy.

B. The Shape of the Income Distribution

A second source of uncertainty in the optimal tax model is the long-term shape of the income distribution and how it varies with the ETI. Forecasting the evolution of aggregate income, much less its distribution, is notoriously difficult, as it requires anticipating major forces—such as technological and cultural changes—whose underlying drivers are poorly understood.

We focus on a narrow slice of the broad uncertainty over the income distribution's shape: the range of possible current underlying income-earning ability distributions that is implied by different plausible values for the ETI. Given an observed income distribution, an existing tax system, and a model of the individual labor supply decision, we can infer a current underlying distribution of income-earning abilities for each value of the ETI.[22]

These current underlying ability distributions are all consistent with one existing income distribution, given the existing tax system, but if the tax system is changed they will translate into different income distributions. That is, when we reverse the inference process that yielded these ability distributions, instead plugging them into the model of individual labor supply with a new tax system, we obtain a new income distribution for each value of the ETI. We can then feed the joint probability distribution of these pairs of elasticities and income distributions into the robust optimal tax model from the previous section.

Formally, consider the individual utility maximization problem: $\max_{y^i} U(c^i, y^i)$, where $c^i = y^i - T(y^i)$, using the same notation as in the linear case above. To relate our results to the large existing literature on optimal taxation without income effects, we assume that the individual utility function takes the Type 1 form from Saez (2001):

$$U(c^i, y^i) = G\left(c^i - \frac{1}{1+(1/\sigma)} \left(\frac{y^i}{w^i} \right)^{1+(1/\sigma)} \right), \tag{30}$$

where G is a concave transformation and the parameter σ controls the ETI and is equal to it when the tax schedule is locally linear.[23] This type of utility function is often used to represent a "constant ETI" economy, and in practice the true ETI is very close to σ even with curvature, so to implement our numerical simulations we set σ equal to the ETI values reported in the survey. The individual's first-order condition (FOC) is therefore

$$w^i = \left(y^i (1 - T'(y^i))^{-\sigma} \right)^{\frac{1}{1+\sigma}}. \tag{31}$$

To obtain $T(y)$, we calibrate the current US tax system using the empirical mapping between a broad measure of market income and consumption, as reported in Piketty, Saez, and Zucman (2018). This measure thus incorporates broad features of the tax and transfer system including not only income taxes at all levels but also other tax credits and phaseouts, in-kind transfers and other social support programs, and an accounting of distributed public goods.

We again use the income distribution from Piketty et al. (2018) to calibrate the status quo income distribution. Using the calibrated status quo tax function T_{US} and the status quo income distribution $H_{T_{US}}(y)$, we can compute an underlying implied ability distribution using the FOC in equation (31) for any assumed value of the parameter σ. More generally, if the policymaker faces an uncertain distribution of possible elasticity values, this procedure permits us to compute a separate underlying ability distribution in each case, which can then be used to compute an optimal expected tax function in the face of such uncertainty. We next compute the results of such an exercise for a range of assumptions about the elasticity of income with respect to tax rates.

We emphasize that this uncertainty represents a narrow slice of the broad uncertainty that exists over the future shape of the income and income-earning ability distributions. The process by which the mean level, much less the variance, of human capital changes over time is poorly understood. And different historical periods have seen dramatically different trends in growth and inequality for which differences in tax policy (that would be captured in our calibration) are unlikely to be the primary, much less only, determining factor. An important task for further research is to incorporate the scope of uncertainty in this dimension of the model, for example, by allowing for a range of paths for the underlying ability distribution as implied by past data.

III. Quantitative Results

In this section, we use the optimality conditions and calibration procedures described above to demonstrate the effect of parameter uncertainty on optimal tax results. In other words, for a given assumption about the average value of a model parameter—in this case, the ETI—what is the effect of uncertainty on the optimal tax schedule?

Here we continue to employ the individual utility function in equation (30). To account for the agent's diminishing marginal utility of consumption (or, equivalently in this model, the policymaker's degree of inequality aversion), an assumption must be made about the shape of the concave transformation G. As Saez (2001) notes, this concavity can be described in a transparent way using "social marginal welfare weights" g^i, proportional to $G'(c^i - (1/(1 + 1/\sigma))(y^i/w^i)^{1+(1/\sigma)})$ at the optimum. In our baseline simulations, we use a logarithmic functional form $G(x) = \ln(x)$.

Note that we focus on uncertainty over the ETI in particular, but in principle one may wish to consider uncertainty over the concave transformation G or over the future income distribution, as discussed above. In addition, there may be uncertainty about whether individuals successfully maximize their own utility or whether they are subject to misperception or other behavioral frictions.[24] We restrict our focus to uncertainty about the ETI to provide a tractable and concrete illustration with new data elicited from economists about the value of this well-known parameter, but we view the extension of uncertainty to other aspects of the model, including its normative features, as an important task for further research.

We first consider a simple example of an economy with two types—a low income-earning ability type and a high income-earning ability type, chosen to correspond to the 25th and 75th percentiles of the US income distribution—and a linear tax system. In this example, we consider an economy in which the planner's subjective distribution of σ is discrete, consisting of just two values: $\sigma = 0.1$ with probability .7, and $\sigma = 1.1$ with probability .3, for an overall expected value of $\sigma = 0.4$. In this setting, we compare the best estimate policy to the robust policy under the two different assumptions about the budget constraint discussed in Section I, where either the tax policy is fully prespecified (with the budget constraint binding in expectation) or the budget must balance in each state by adjusting the lump-sum grant. The optimal linear tax rates and lump-sum grants for these specifications are reported in table 1. From this simple example, we note three features of the optimal tax regimes: (i) robustness generally increases tax rates and lump-sum grants, resulting in a more progressive tax system; (ii) enforcing within-state budget constraints generally decreases tax rates, reducing the progressive effect of robustness; and (iii) state-specific lump-sum grants are decreasing in the elasticity as income and, consequently, revenue decline more in those states.[25]

We next perform simulations in a more complex setting, where we compute the optimal fully nonlinear tax. Details of the implementation are discussed in Section C. In these simulations, we calibrate the economy at the status quo with the modern US income and consumption distributions. We revisit the distribution of states considered above: $\sigma = 0.1$ with probability .7, and $\sigma = 1.1$ with probability .3. Features of the optimal best

Table 1
Linear Tax Illustration with Two Types

		t (%)	b	
1	Status quo	16.9	$12,848	
2	Best estimate, fully prespecified	33.9	$13,982	
3	Robust, fully prespecified	35.9	$14,741	
4	Best estimate, within-state balance	33.9	$14,975 ($\sigma = .1$)	$11,914 ($\sigma = 1.1$)
5	Robust, within-state balance	34.9	$15,426 ($\sigma = .1$)	$12,072 ($\sigma = 1.1$)

Note: This table reports the status quo and optimal linear tax rates under a given average value of the structural elasticity parameter σ, but for different assumptions about uncertainty, in a simple setting with two income-earning ability types. In the status quo, the low-ability type earns the 25th percentile income from the modern US income distribution, whereas the high-ability type earns the 75th percentile income. Each type's consumption is equal to the empirical consumption that corresponds to its income, adjusted by the same lump sum to ensure the budget is balanced in the status quo and the exogenous revenue requirement is zero given a linear tax system. Each type constitutes 50% of the population. In row 1, the status quo linear tax t and lump-sum grant b are reported. In row 2, the optimal linear tax under the policymaker's best estimate of $\sigma = .4$ and a budget constraint that binds in expectation is reported with a prespecified grant. In row 3, the robust optimal linear tax under the policymaker's belief that there is a 70% chance that $\sigma = .1$ and a 30% chance that $\sigma = 1.1$ and a budget constraint that binds in expectation is reported with a prespecified grant. In row 4, the optimal linear tax under the policymaker's best estimate of $\sigma = .4$ and within-state budget constraints is reported with state-specific grants. In row 5, the robust optimal linear tax under the same beliefs as in row 3 and within-state budget constraints is reported with state-specific grants.

estimate and robust tax policies are plotted in figure 3. Figure 3*a* displays the optimal schedule of marginal tax rates computed under the best estimate specification, comparing it to the fully prespecified robust tax schedule and the within-state budget-balancing robust tax schedule. Because marginal tax rates are prespecified in each setting, these schedules are

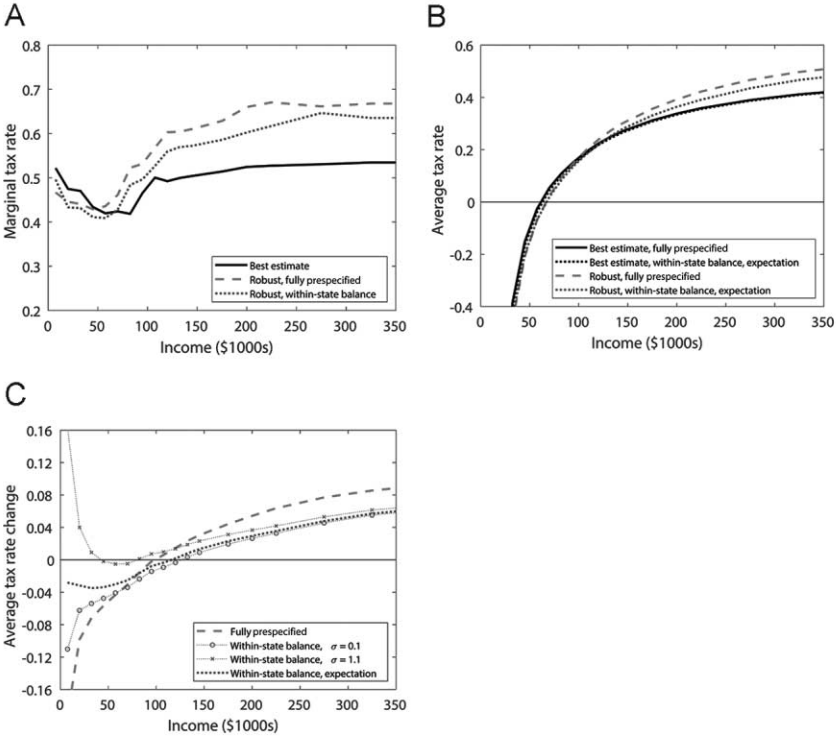


Fig. 3. Optimal tax schedules under parameter uncertainty. (*a*) The first graph plots optimal marginal tax rates under a given average value of the structural elasticity parameter σ, but for different assumptions about uncertainty. The line labeled "Best estimate" plots the tax schedule optimized under the certain value σ = 0.4. In the "fully prespecified" case, the lump-sum grant is specified so that the budget constraint binds when σ = 0.4 and is $27,201. In the "within-state balance" case, a lump-sum grant is specified in each of two elasticity states, σ = 0.1 and σ = 1.1, ensuring within-state budget constraints bind. The grant received is $31,426 if σ = 0.1 and $19,650 if σ = 1.1. The lines labeled "Robust" plot the robust optimal tax schedule when the policymaker is uncertain about the elasticity of taxable income and either fully prespecifies the tax or ensures within-state balance by satisfying within-state budget constraints. The policymaker believes there is a 70% chance that σ = 0.1 and a 30% chance that σ = 1.1. The prespecified lump-sum grant is $28,239. The state-specific grant is $31,929 if σ = 0.1 and $18,105 if σ = 1.1. (*b*) The second graph plots the optimal average fully prespecified tax rates and the expectation of the average tax rates with within-state balance. (*c*) The third graph plots the differences in optimal average tax rates between the "Best estimate" and "Robust" specifications. A color version of this figure is available online.

not sensitive to the ETI that actually obtains. Figure 3*b* plots the schedule of average tax rates in each setting. In the within-state budget balance setting, the lump-sum grant depends on which ETI turns out to obtain, and so the schedule of average tax rates in that setting is state dependent. To retain legibility in the plot, we display only the expected schedule of average tax rates rather than the schedule in each state. Finally, figure 3*c* plots the difference in average tax rates under the robust policy relative to the best estimate policy, in both the fully prespecified setting and the within-state budget balance setting. (For the latter case, we also plot the difference in average tax rates within each state.)

To explore the impact of even greater skewness in the subjective distribution, figure 4 plots results for a subjective distribution in which σ takes the values of 0.1, 0.2, and 2.6 with probabilities .4, .5, and .1, respectively, resulting in the same average value of $\sigma = 0.4$.

We note two patterns that are apparent from these simulations with a fully nonlinear tax.

First, the shape of the optimal tax schedule remains broadly similar with and without robustness, generally retaining the trademark U-shape discussed in Diamond (1998) and Saez (2001). The shape changes more as the distribution of σ becomes more skewed, as demonstrated in figure 4, where the U-shape spans a narrower range of incomes under the robust optimal tax schedules. Under both distributions of σ, the tax schedules under within-state budget constraints depart less strongly from the certainty benchmark. In some cases, the shape of the robust optimal tax schedule appears somewhat smoother than the best estimate tax. For example, the "jaggedness" of the best estimate tax schedule in figure 3*a*, just below \$50,000 and \$100,000, appears less pronounced in the robust schedules. This smoothness is directly tied to the policy's recognition of uncertainty. If there is a concentration of the population density at a specific point in the income distribution, and thus income-earning ability distribution, the optimal tax schedule under certainty will feature a dip at the point of that higher density to reduce distortionary marginal tax rates for that high density of workers. However, if the elasticity is uncertain, the location of that point of population density under a reformed tax schedule is less predictable and so the depressing effects on optimal tax rates are more diffuse. In figure 3, this feature is visible at incomes below \$100,000, where a large majority of the population is concentrated in each state. This result, however, does not always obtain: for instance, under the high degree of skewness in the distribution of σ in figure 4, this smoothness is masked by a steep increase in rates across incomes above \$50,000 driven by the large probability placed on low values of σ.

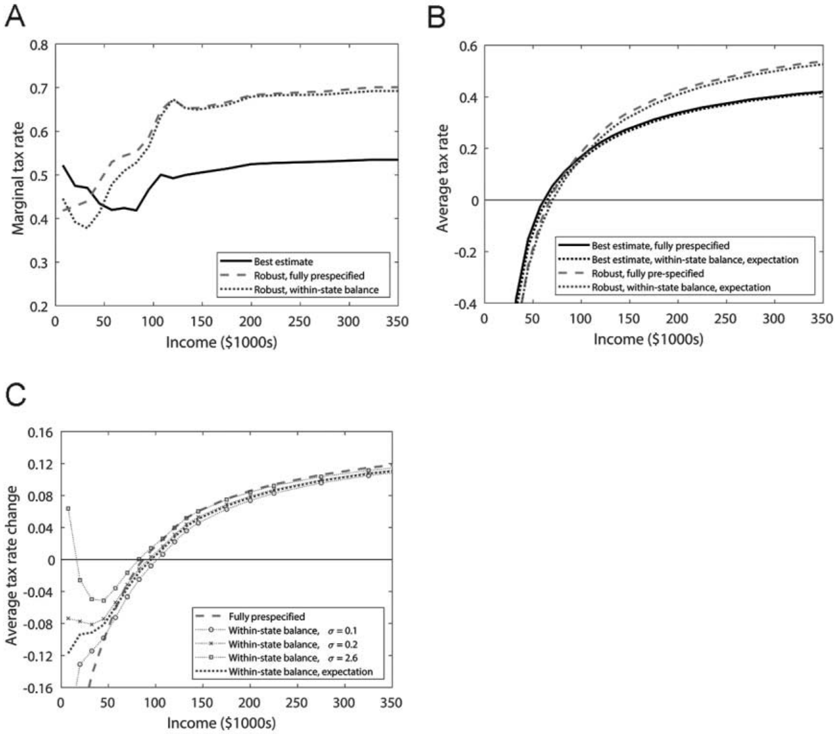


Fig. 4. Optimal tax schedules under parameter uncertainty (skewed). (*a*) The first graph plots optimal marginal tax rates under a given average value of the structural elasticity parameter σ, but for different assumptions about uncertainty. The line labeled "Best estimate" plots the tax schedule optimized under the certain value σ = 0.4. In the "fully prespecified" case, the lump-sum grant is specified so that the budget constraint binds when σ = 0.4 and is $27,201. In the "within-state balance" case, a lump-sum grant is specified in each of three elasticity states, σ = 0.1, σ = 0.2, and σ = 2.6, ensuring within-state budget constraints bind. The grant received is $31,426 if σ = 0.1, $29,933 if σ = 0.2, and $9,978 if σ = 2.6. The lines labeled "Robust" plot the robust optimal tax schedule when the policymaker is uncertain about the elasticity of taxable income and either fully prespecifies the tax or ensures within-state balance by satisfying within-state budget constraints. The policymaker believes there is a 40% chance that σ = 0.1, a 50% chance that σ = 0.2, and a 10% chance that σ = 2.6. The prespecified lump-sum grant is $29,671. The state-specific grant is $32,381 if σ = 0.1, $29,813 if σ = 0.2, and $8,827 if σ = 2.6. (*b*) The second graph plots the optimal average fully prespecified tax rates and the expectation of the average tax rates with within-state balance. (*c*) The third graph plots the differences in optimal average tax rates between the "Best estimate" and "Robust" specifications. A color version of this figure is available online.

The second key pattern from figures 3 and 4 is that the robust tax schedule is more progressive. This pattern is particularly apparent from the graphs in figures 3*c* and 4*c*: the lines sloping upward is an indication that average tax rates are higher for high earners relative to low earners under robustness. This "progressivity effect" has a somewhat subtle source. Starting with a given tax policy, states in which the elasticity σ is higher will have income distributions that are shifted downward relative to those with lower elasticities. High-elasticity states are also those in which taxes are particularly distortionary, so optimal marginal tax rates in those states are low. Thus, a policy designed to apply across all states will account for the covariance between these parameters and optimally impose lower marginal tax rates at lower incomes and higher rates at higher incomes, yielding an optimal tax rate schedule that is more progressive than under the best estimate tax policy. An illuminating subtlety is that when the budget constraint binds in each state—and thus resources cannot be moved across states—the realized robust optimal tax policy is not always more progressive, as seen in figure 3 when $\sigma = 1.1$. Due to the lower levels of tax revenue and consequently smaller lump-sum grants generated in higher-elasticity states, the tax policy optimized under uncertainty is less progressive when realized in those states. Nevertheless, the tax schedule remains generally more progressive in expectation under uncertainty even when the budget constraint must be satisfied in each state.

Although the examples in figures 3 and 4 are illustrative, for our preferred estimates of the robust tax policy, we rely on our survey data, discussed in Subsection II.A, elicited from public finance economists. Figure 2 plots the ETI distribution obtained by averaging those distributions, and figure 5 plots the optimal tax schedules generated using this distribution. Specifically, we assume the parameter σ may take each of six different values corresponding to intervals we specified in the survey (0.10, 0.20, 0.40, 0.75, 1.50, and 2.50), with probabilities corresponding to the average across respondents (normalized to ensure a sum of 100%). This procedure results in a less skewed distribution of σ than that which we assumed earlier. As a consequence, although the robust optimal income tax policy remains more progressive and better for welfare than the best estimate optimal policy, it is also more similar to the best estimate policy than in figures 3 and 4.

In each setting, we estimate the expected annual money-metric welfare gain from replacing the best-guess optimum with the robust optimal income tax (under the same budget constraint assumption). The welfare gain is higher when the budget constraint binds in expectation—allowing for welfare-increasing transfers of resources across states—than it is when budget constraints bind in each state. Under our preferred calibration using our survey of economists, the gains are the most modest at 1.1% and 0.2% of consumption, or \$191 billion and \$54 billion, under the constraints that bind in expectation and each state, respectively.[26]

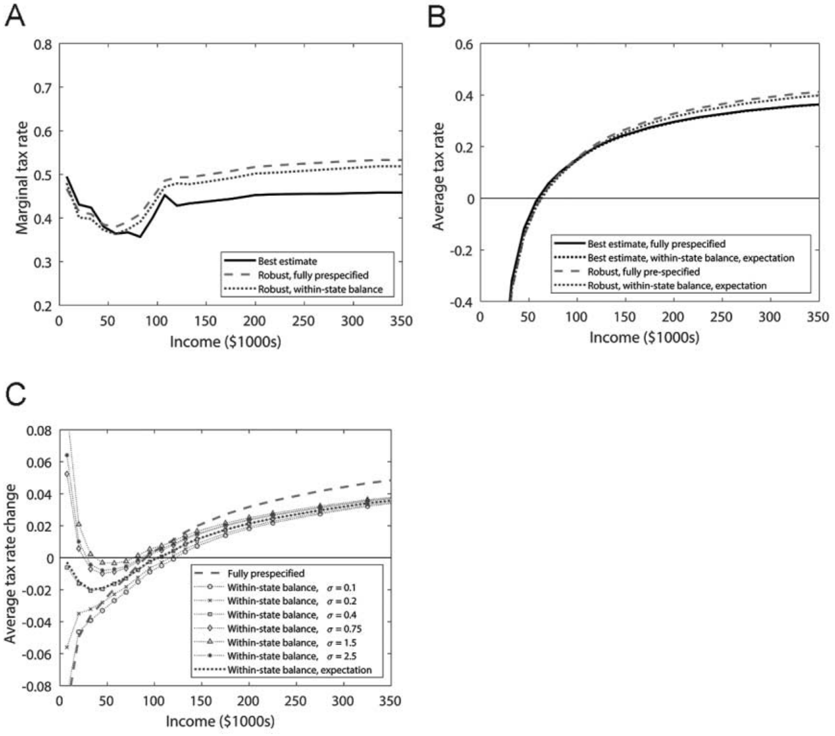


Fig. 5. Optimal tax schedules under parameter uncertainty (pilot survey data). (*a*) The first graph plots optimal marginal tax rates under a given average value of the structural elasticity parameter σ, but for different assumptions about uncertainty. The line labeled "Best estimate" plots the tax schedule optimized under the certain value σ = 0.57, the expected value of the elasticity of taxable income from our survey of economists. In the "fully prespecified" case, the lump-sum grant is specified so that the budget constraint binds when σ = 0.57 and is $23,817. In the "within-state balance" case, a lump-sum grant is specified in each elasticity state for which we elicited beliefs in our survey, ensuring within-state budget constraints bind. The grant received is $28,140 if σ = 0.10, $27,144 if σ = 0.20, $25,273 if σ = 0.40, $22,342 if σ = 0.75, $17,214 if σ = 1.50, and $12,211 if σ = 2.50. The lines labeled "Robust" plot the robust optimal tax schedule when the policymaker is uncertain about the elasticity of taxable income and either fully prespecifies the tax or ensures within-state balance by satisfying within-state budget constraints. The policymaker has the average belief of the economists from our survey, where there is a 11.7% chance that σ = 0.10, a 21.3% chance that σ = 0.20, a 33.8% chance that σ = 0.40, a 21.0% chance that σ = 0.75, a 8.4% chance that σ = 1.50, and a 3.8% chance that σ = 2.50. The prespecified lump-sum grant is $24,248. The state-specific grant is $28,536 if σ = 0.10, $27,311 if σ = 0.20, $25,067 if σ = 0.40, $21,697 if σ = 0.75, $16,266 if σ = 1.50, and $11,478 if σ = 2.50. (*b*) The second graph plots the optimal average fully prespecified tax rates and the expectation of the average tax rates with within-state balance. (*c*) The third graph plots the differences in optimal average tax rates between the "Best estimate" and "Robust" specifications. A color version of this figure is available online.

Although using the robust optimal policy increases welfare in expectation both in the fully prespecified setting and when the budget constraint binds in each state, in the latter setting the gains are unequal across states, and using the robust optimal policy may even decrease welfare when the ETI turns out to be very high. Figure 6 plots the welfare gain (or loss) in each state under our baseline calibration. When the budget constraint binds in expectation, the welfare effect of adopting the robust policy is positive in every state and increases monotonically in the elasticity. This increase reflects the fact that the more progressive robust policy results in a higher lump-sum grant, which generates particularly large gains in the high ETI states where the marginal value of consumption is higher. Under

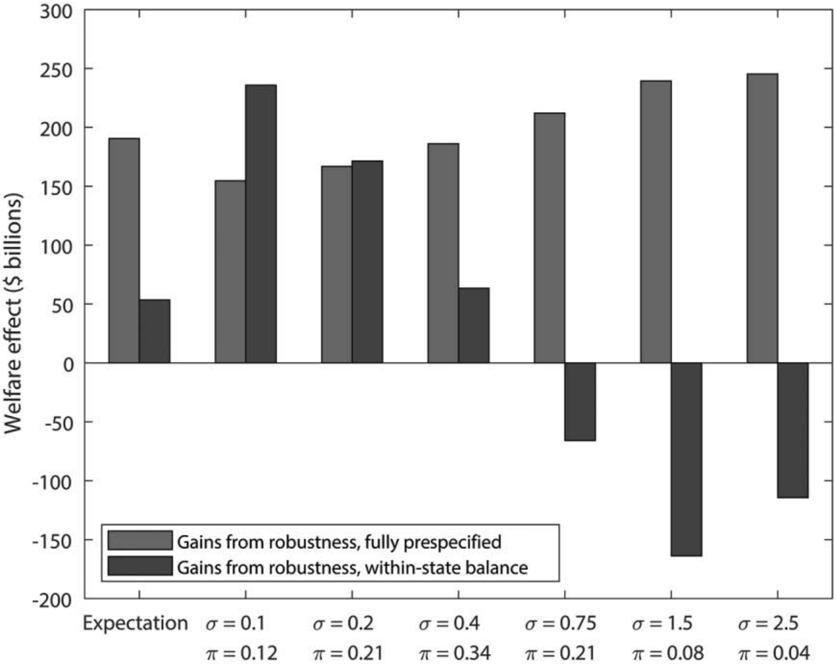


Fig. 6. Welfare gains from robustness (pilot survey data). This figure plots the estimated annual money-metric welfare effect of switching from the best estimate income tax policy to the robust optimal income tax policy when each state of σ—with associated probability π—obtains and in expectation. The policymaker has the average belief about σ of the academic economists from our survey. The fully prespecified best estimate policy, with a lump-sum grant specified so that the budget constraint binds when σ is equal to its best estimate, is compared with the fully prespecified robust optimal policy with a lump-sum grant specified so that the budget constraint binds in expectation. The best estimate policy, with state-specific lump-sum grants specified so that budget constraints bind within each state, is compared with the robust optimal policy with within-state budget balance. See Subsection C.2 for implementation details. A color version of this figure is available online.

within-state budget constraints, the lump-sum grant is larger in low-elasticity states and smaller in high-elasticity states under the robust policy, causing the welfare effect to generally decrease in the elasticity. However, due to the crossing of the best estimate and robust marginal tax rate schedules around $50,000 in income, the lower marginal tax rates under the robust policy below that threshold, and the downward-shifting of the income distribution as the elasticity increases, the change in revenue from adopting the robust policy does not necessarily monotonically decrease in the elasticity. As a result, the decrease in the lump-sum grant from adopting the robust policy is greater when $\sigma = 1.5$ than in the highest-elasticity state. Consequently, the welfare losses from robustness are estimated to be the largest in that state. These potential welfare losses in higher-elasticity states result in lower expected welfare gains under within-state budget constraints than under the constraint that binds in expectation.

IV. Conclusion

In this paper, we propose a method by which to design economic policy analyses for robustness to uncertainty over the models and parameters at their core. We apply this method to the topic of optimal income taxation in the Mirrleesian tradition and use both existing and novel empirical evidence on the range of professional economists' beliefs over the value of the ETI to introduce uncertainty into the analysis. We analytically and quantitatively characterize robust optimal income taxes and compare them to benchmark results, suggesting that this method might be fruitfully applied to a wide range of economic policies.

Appendix

A. Derivations with Direct Maximization

In this appendix, we show how the results derived for the optimal linear tax through the perturbation method can also be obtained through a direct maximization of the tax authority's objective.

A.1. Certainty Benchmark

Individual utility is $U^i(c, y) = c - v^i(y/w^i)$, with v^i increasing and convex. Types are indexed by i with population fractions p^i. The social welfare function is $\Sigma_i p^i \Phi(U^i(c, y))$, with Φ increasing and concave. Tax policy is defined by a linear rate t, which determines a lump-sum grant $b(t) = t\sum_i p^i y^i$. Individuals choose $y^i(t) = \arg\max_y\{y(1-t) + b(t) - v^i(y/w^i)\}$, and we denote $\zeta^i = (dy^i(t)/d(1-t)) \cdot ((1-t)/y^i)$.

The tax authority's policy problem is $\max_t\{\Sigma_i p^i \Phi(y^i(t)(1-t) + b(t) - v^i(y^i(t)/w^i))\}$. The FOC for the optimum is

$$\sum_i p^i \Phi'(U^i)\left(-y^i(t) + \sum_j p^j y^j(t) - t\sum_j p^j \frac{dy^j(t)}{d(1-t)}\right) = 0, \tag{A1}$$

or, rearranging,

$$\frac{t}{1-t}\sum_i p^i \Phi'(U^i) \cdot \sum_j p^j \zeta^j y^j = \sum_i p^i \Phi'(U^i) \cdot \sum_j p^j y^j - \sum_i p^i \Phi'(U^i) y^i. \tag{A2}$$

Now, we add notation for simplicity: $\lambda = \Sigma_i p^i \Phi'(U^i)$ is the social value of marginally augmenting the lump-sum grant (also known as the MVPF); $\bar{x} = \Sigma_i p^i(x^i)$ is the expected value of an individual-specific variable x^i; and $g^i = \Phi'(U^i)/\lambda$ is the marginal social welfare weight on type i. Note that $\bar{g} = \Sigma_i p^i(\Phi'(U^i)/\lambda) = (\Sigma_i p^i \Phi'(U^i)/\Sigma_j p^j \Phi'(U^j)) = 1$. With these terms, we can rewrite the FOC as

$$\frac{t}{1-t} = \frac{E[y^i] - E[g^i y^i]}{E[\zeta^i y^i]} \tag{A3}$$

$$= \frac{-\mathrm{Cov}[g^i, y^i]}{E[\zeta^i y^i]}. \tag{A4}$$

If we additionally assume $\zeta^i \equiv \bar{\zeta}$ is constant, then this FOC simplifies further to

$$\frac{t}{1-t} = \frac{-\mathrm{Cov}\left[g^i, \frac{y^i}{\bar{y}}\right]}{\bar{\zeta}}. \tag{A5}$$

A.2. Uncertainty, State-Invariant Lump-Sum Grant

In this scenario, states are indexed by s, with associated probabilities π_s. Individual utility is $U_s^i(c, y) = c - v_s^i(y/w_s^i)$; population fractions are p_s^i. The social welfare function is $\Sigma_s \pi_s \Sigma_i p_s^i \Phi(U_s^i(c, y))$, with Φ increasing and concave. Tax policy is made up of a state-invariant linear rate t, which determines the state-invariant lump-sum grant: $b(t) = t\Sigma_s \pi_s \Sigma_i p_s^i y_s^i$. Note that although this setup implies resources can be transferred across states, it is not the same as imposing the optimal state-specific lump-sum grants, as that would generally entail making the grant higher in states where labor disutility $v_s^i(y/w_s^i)$ is high on average. In other words, this is the optimal tax and transfer when both must be prespecified.

The individual optimization problem is $y_s^i(t) = \arg\max_y\{y(1-t) + b(t) - v_s^i(y/w_s^i)\}$, where we denote $\zeta_s^i = (dy_s^i(t)/d(1-t)) \cdot ((1-t)/y_s^i)$. The tax authority's policy problem is

$$\max_t \left\{ \sum_s \pi_s \sum_i p_s^i \Phi(y_s^i(t)(1-t) + b(t) - v_s^i(y_s^i(t)/w_s^i)) \right\}. \tag{A6}$$

The optimum is characterized by the following first-order condition:

$$\sum_s \pi_s \sum_i p_s^i \Phi'(U_s^i) \left(-y_s^i(t) + \sum_{s'} \pi_{s'} \sum_j p_{s'}^j y_{s'}^j - t \sum_{s'} \pi_{s'} \sum_j p_{s'}^j \frac{dy_{s'}^j(t)}{d(1-t)} \right) = 0. \tag{A7}$$

Again, we introduce notation, now adapted for uncertainty: $\lambda = \Sigma_s \pi_s \Sigma_i p_s^i \Phi'(U_s^i)$ is the MVPF; $\bar{x} = \Sigma_s \pi_s \Sigma_i p_s^i x_s^i$ denotes the expectation of a variable x over types and states; $\bar{x}_s = \Sigma_i p_s^i x_s^i$ denotes the average across types within a state s; $g_s^i = (\Phi'(U_s^i)/\lambda)$, still implying $\bar{g} = 1$. With these in hand, we rewrite the optimality condition as

$$\frac{t}{1-t} = \frac{E[y_s^i] - E[g_s^i y_s^i]}{E[\zeta_s^i y_s^i]} \tag{A8}$$

$$= \frac{-\mathrm{Cov}[g_s^i, y_s^i]}{E[\zeta_s^i y_s^i]}. \tag{A9}$$

And if we further assume that $\zeta_s^i \equiv \bar{\zeta}_s$, that is, there is uncertainty about the (homogeneous) value of ζ, then the FOC simplifies to

$$\frac{t}{1-t} = \frac{-\mathrm{Cov}[g_s^i, y_s^i]}{\bar{\zeta}\bar{y} + \mathrm{Cov}[\bar{\zeta}_s, \bar{y}_s]} \tag{A10}$$

$$= \frac{-\mathrm{Cov}\left[g_s^i, \frac{y_s^i}{\bar{y}}\right]}{\bar{\zeta}\left(1 + \mathrm{Cov}\left[\frac{\bar{\zeta}_s}{\bar{\zeta}}, \frac{y_s^i}{\bar{y}}\right]\right)}. \tag{A11}$$

A.3. Uncertainty, Budget Constraint Binds in Each State

Finally, we assume that the budget constraint binds within each state; that is, $b_s(t) = t\Sigma_i p_s^i y_s^i$. The policy problem is $\max\{\Sigma_s \pi_s \Sigma_i p_s^i \Phi(y_s^i(t)(1-t) + b_s(t) - v_s^i(y_s^i(t)/w_s^i))\}$. Because the budget constraint applies in each state, we define the marginal value of public funds within each state as $\lambda_s = \sum_i p_s^i \Phi'(U_s^i)$.

The optimality condition is

$$\sum_s \pi_s \sum_i p_s^i \Phi'(U_s^i) \left(-y_s^i(t) + \sum_j p_s^j y_s^j - t \sum_j p_s^j \frac{dy_s^j(t)}{d(1-t)} \right) = 0. \quad \text{(A12)}$$

We can rewrite this result as

$$\frac{t}{1-t} \sum_s \pi_s \lambda_s \sum_i p_s^i \zeta_s^i y_s^i = \sum_s \pi_s \lambda_s \sum_i p_s^i y_s^i - \sum_s \pi_s \lambda \sum_i p_s^i g_s^i y_s^i, \quad \text{(A13)}$$

implying

$$\frac{t}{1-t} = \frac{E[\lambda_s y_s^i] - \lambda E[g_s^i y_s^i]}{E\left[\lambda_s \zeta_s^i y_s^i\right]} \quad \text{(A14)}$$

$$= \frac{-\mathrm{Cov}[g_s^i, y_s^i] + \mathrm{Cov}[(\lambda_s/\lambda), y_s^i]}{E[\zeta_s^i y_s^i] + \mathrm{Cov}[(\lambda_s/\lambda), \zeta_s^i y_s^i]} \quad \text{(A15)}$$

And if we further assume that $\zeta_s^i \equiv \bar{\zeta}_s$, that is, there is uncertainty about the (homogeneous) value of ζ, then the FOC simplifies to

$$\frac{t}{1-t} = \frac{-\mathrm{Cov}[g_s^i, y_s^i] + \mathrm{Cov}[(\lambda_s/\lambda), y_s^i]}{E[\bar{\zeta}_s \bar{y}_s] + \mathrm{Cov}[(\lambda_s/\lambda), \bar{\zeta}_s \bar{y}_s]} \quad \text{(A16)}$$

$$= \frac{-\mathrm{Cov}[g_s^i, (y_s^i/\bar{y})] + \mathrm{Cov}[(\lambda_s/\lambda), (y_s^i/\bar{y})]}{\bar{\zeta} + \mathrm{Cov}[\bar{\zeta}_s, (\bar{y}_s/\bar{y})] + \mathrm{Cov}[(\lambda_s/\lambda), \bar{\zeta}_s(\bar{y}_s/\bar{y})]} \quad \text{(A17)}$$

$$= \frac{-\mathrm{Cov}[g_s^i, (y_s^i/\bar{y})] + \mathrm{Cov}[(\lambda_s/\lambda), (y_s^i/\bar{y})]}{\bar{\zeta}(1 + \mathrm{Cov}[(\bar{\zeta}_s/\bar{\zeta}), (\bar{y}_s/\bar{y})] + \mathrm{Cov}[(\lambda_s/\lambda), ((\bar{\zeta}_s \bar{y}_s)/(\bar{\zeta}\bar{y}))])}. \quad \text{(A18)}$$

B. Derivations of Nonlinear Optimal Tax Formulas

In this appendix, we show how to obtain the results derived for the optimal nonlinear tax through the perturbation method.

B.1. Certainty Benchmark

Individual utility is $U^\theta(c, y) = c - v^\theta(y/w^\theta)$, with v increasing and convex. Types are continuously indexed by $\theta \in \Theta$ with distribution $F(\theta)$ and associated density $f(\theta)$. The social welfare function is $\int \Phi(U^\theta(c, y))\, dF(\theta)$, with Φ increasing and concave. Tax policy consists of a nonlinear income tax $T(y)$, which consists of marginal tax rates $T'(y)$ and a lump-sum grant $-T(0)$ equal to per capita tax revenues. Individuals choose $y^\theta(T)$ = arg

$\max_y\{y - T(y) - v^\theta(y/w^\theta)\}$, resulting in the endogenous income distribution $H_T(y)$ and associated density $h_T(y)$. We denote the compensated labor supply elasticity as $\zeta(y) = (\partial y/\partial(1 - T'(y))) \cdot ((1 - T'(y))/y)$ and have income effect $\eta(y) \equiv 0$.

The tax authority's policy problem is $\max_T\{\int \Phi(U^\theta(y^\theta - T(y^\theta) - v^\theta(y^\theta/w^\theta)))dF(\theta)\}$ subject to the governmental budget constraint $\int T(y)h_T(y)dy \geq R$, where R is an exogenous revenue requirement, and subject to each y^θ solving the individual's optimization problem. We add the following notation for simplicity: $g(y) = \Phi'(U^\theta(c, y))/\lambda$ is the marginal social welfare weight on y-earners of type θ—assuming a one-to-one mapping between θ and y—and

$$\begin{aligned}\lambda &= \int_\Theta \Phi'(U^\theta)dF(\theta) - \lambda\int_0^\infty \frac{T'(y)}{1 - T'(y)}\eta(y)dH_T(y) \\ &= \frac{\displaystyle\int_\Theta \Phi'(U^\theta)dF(\theta)}{1 + \displaystyle\int_0^\infty \frac{T'(y)}{1 - T'(y)}\eta(y)dH_T(y)}\end{aligned} \tag{A19}$$

is the MVPF, which reduces to $\lambda = \int \Phi'(U^\theta)dF(\theta)$ in the case with no income effects. We can further define $\hat{g}(y) = g(y) - (T'(y)/(1 - T'(y)))\eta(y)$ as the social marginal utility of consumption, accounting for fiscal externalities from income effects, in which case we have $\hat{g}(y) = g(y)$ when income effects are absent. Dividing equation (A19) by λ demonstrates that $\int_0^\infty \hat{g}(y)dH_T(y) = 1$.

We use the perturbation method to derive the FOC for optimality of the tax policy, considering a small increase in the marginal tax rate $d\tau$ in an interval of size ε around some earnings level y^*. The mechanical effect of this reform is to raise revenue from individuals with $y > y^*$ and thereby reduce the after-tax income of those individuals, at a welfare cost of $\hat{g}(y)$:

$$dM = \varepsilon d\tau \int_{y^*}^\infty (1 - \hat{g}(z))h_T(z)dz. \tag{A20}$$

The behavioral effect of the reform corresponds to the negative fiscal externality that results from those individuals reducing their effort, with

$$dB = \varepsilon d\tau \frac{-T'}{1 - T'}\zeta(y^*)y^*h_T(y^*). \tag{A21}$$

At the optimum, $dM + dB = 0$, yielding the FOC:

$$\frac{T'(y)}{1-T'(y)} = \frac{1}{y\zeta(y)h_T(y)}\int_y^\infty (1-\hat{g}(z))h_T(z)dz. \tag{A22}$$

B.2. Uncertainty: Fully Prespecified Robust Optimum

Now we have states indexed by s with associated probabilities π_s. Individual utility is $U_s^\theta(c, y) = c - v_s^\theta(y/w_s^\theta)$. The individual-type distribution in each state is denoted $F_s(\theta)$, with associated density $f_s(\theta)$. Tax policy consists of a state-invariant nonlinear income tax $T(y)$, defined by a schedule of marginal tax rates $T'(y)$ and a prespecified lump-sum grant $-T(0)$, equal to expected per capita tax revenues. In each possible state, individuals choose $y_s^\theta(T) = \arg\max_y\{y - T(y) - v_s^\theta(y/w_s^\theta)\}$, resulting in the endogenous income distribution $H_{T,s}(y)$ and associated density $h_{T,s}(y)$. We denote the state-specific compensated labor supply elasticity as $\zeta_s(y) = (\partial y_s/\partial(1-T'(y_s)))\cdot(1-T'(y_s)/y_s)$.

In this setting, the tax authority's policy problem is

$$\max_T\left\{\sum_s \pi_s \int_\Theta \Phi\left(\left(U_s^\theta(y_s^\theta - T(y_s^\theta) - v_s^\theta(y_s^\theta/w_s^\theta)\right)\right)dF_s(\theta)\right\} \tag{A23}$$

subject to a government resource constraint $\Sigma_s \pi_s[\int T(y)h_{s,T}(y)dy] \geq R$, and subject to each y_s^θ solving the individual's optimization problem above. The resource constraint binds in expectation, implying that resources can be transferred across states. Thus the marginal value of public funds is given by

$$\begin{aligned}\lambda &= \sum_s \pi_s \int_\Theta \Phi'(U_s^\theta)dF(\theta) - \lambda \sum_s \pi_s \int_0^\infty \frac{T'(y)}{1-T'(y)}\eta(y)dH_T(y) \\ &= \frac{\sum_s \pi_s \int_\Theta \Phi'(U_s^\theta)dF(\theta)}{1 + \sum_s \pi_s \int_0^\infty \frac{T'(y)}{1-T'(y)}\eta(y)dH_T(y)}\end{aligned} \tag{A24}$$

and we normalize the social marginal utility of consumption by λ to obtain the marginal social welfare weights $g_s(y) = \Phi'(U_s^\theta(c, y))/\lambda$. With $\hat{g}_s(y) = g_s(y) - (T'(y)/(1-T'(y)))\eta_s(y)$, dividing equation (A24) by λ yields $\Sigma_s \pi_s \int_0^\infty \hat{g}_s(y)dH_{T,s}(y) = 1$. As in the linear case, this full prespecified policy regime implies that the tax authority can costlessly transfer resources across states, effectively using the lump-sum grant to provide some insurance.

Using the perturbation method to derive the FOC for optimality of the tax policy, we again consider a small increase in the marginal tax rate $d\tau$ in an interval of size ε around y^*. The expected mechanical effect of this reform is to raise revenue from individuals in each state with $y > y^*$ and thereby reduce their consumption:

$$dM^e = \varepsilon d\tau \sum_s \pi_s \left[\int_{y^*}^{\infty} (1 - \hat{g}_s(z))\, h_{s,T}(z) dz \right]. \tag{A25}$$

The expected behavioral effect is the expected negative fiscal externality that results from those individuals reducing their effort:

$$dB^e = \varepsilon d\tau \sum_s \pi_s \left[\frac{-T'(y^*)}{1 - T'(y^*)} \zeta_s(y^*) y^* h_{s,T}(y^*) \right]. \tag{A26}$$

We set $dM^e + dB^e = 0$, yielding the FOC at the optimum:

$$\frac{T'(y)}{1 - T'(y)} = \frac{\sum_s \pi_s \left[\int_{y}^{\infty} (1 - \hat{g}_s(z))\, h_{s,T}(z) dz \right]}{y \sum_s \pi_s [\zeta_s(y) h_{s,T}(y)]}. \tag{A27}$$

This corresponds to the equation in the text.

B.3. Uncertainty, Budget Constraint Binds in Each State

Here we assume that the government budget constraint binds within each state, with the lump-sum grant adjusting to balance the budget. Formally, the state-dependent tax policy is defined as $\mathcal{T}_s(y) = T(y) - b_s$, where $T(0) = 0$ and b_s is the state-dependent lump-sum grant, with $b_s = \int_0^\infty T(y) h_{s,T}(y) dy$ for each s; marginal tax rates $T'(y)$ remain state invariant. The policy problem remains the same as in equation (A23), except that the budget constraint now applies in each state (i.e., $\int \mathcal{T}_s(y) h_{s,T}(y) dy \geq R$). In this setting, resources cannot be transferred across states and the social value of raising b_s differs across states. We define the state-specific MVPF as

$$\begin{aligned} \lambda_s &= \int_\Theta \Phi'(U_s^\theta) dF(\theta) - \lambda_s \int_0^\infty \frac{T'(y)}{1 - T'(y)} \eta_s(y) dH_{s,T}(y) \\ &= \frac{\int_\Theta \Phi'(U_s^\theta) dF(\theta)}{1 + \int_0^\infty \frac{T'(y)}{1 - T'(y)} \eta_s(y) dH_{s,T}(y)}. \end{aligned} \tag{A28}$$

Using the perturbation method and considering a small increase in the marginal tax rate $d\tau$ in an interval of size ε around y^*, we note that the revenue generated in each state must be weighted by $\frac{\lambda_s}{\lambda}$, accounting for the fact that funds may be more or less valuable in state s than they are in expectation overall. As a result, we have the modified expected mechanical effect:

$$dM^e = \varepsilon d\tau \sum_s \pi_s \left[\int_{y^*}^{\infty} \left(\frac{\lambda_s}{\lambda} - \hat{g}_s(z) \right) h_{s,T}(z) dz \right]. \tag{A29}$$

Similarly, we weight the behavioral effect in each state by the MVPF because the negative fiscal externality is most costly in states where λ_s is highest, resulting in the expected behavioral effect:

$$dB^e = \varepsilon d\tau \sum_s \pi_s \frac{\lambda_s}{\lambda} \left[\frac{-T'(y^*)}{1 - T'(y^*)} \zeta_s(y^*) y^* h_{s,T}(y^*) \right]. \tag{A30}$$

We again derive the FOC at the optimum by setting $dM^e + dB^e = 0$, yielding

$$\frac{T'(y)}{1 - T'(y)} = \frac{E_s \left[\int_{y}^{\infty} (\lambda_s - \hat{g}_s(z)\lambda) h_{s,T}(z) dz \right]}{y E_s [\lambda_s \zeta_s(y) h_{s,T}(y)]}. \tag{A31}$$

C. Documentation of Simulations

We simulate robust optimal nonlinear tax policies for the United States under various beliefs about the long-term ETI using the fixed-point algorithm detailed in Subsection C.1. We implement the algorithm and estimate the welfare effects of changing tax policies in MATLAB using the files detailed in Subsection C.3.

C.1. Details of the Fixed-Point Algorithm

The algorithm is implemented in the following general steps, with further details provided below:

1. Characterize beliefs about the long-term ETI as vectors of elasticity states and probabilities assigned to each state.
2. Import income and consumption distribution data to infer an underlying income-earning ability distribution for each state, calibrate the

status quo tax system, and determine the policymaker's revenue requirement. Initialize the income and consumption distributions in each state at the status quo and solve for state-specific labor disutility constants that ensure that the utility of each individual is equal across states in the status quo.

3. Compute the marginal social welfare weight for each ability type in each state. Update the marginal tax rate schedule with an alternative tax schedule obtained from the policymaker's FOC, either in the case in which the budget constraint binds in expectation or the case in which it binds in each state.

4. Update the income distribution in each state with the optimal labor supply choice under the updated marginal tax rate schedule using the FOC in the individual's utility maximization problem. Repeat this step until the change in labor supply choices becomes trivially small.

5. Update the lump-sum grant, either within each state (for the specification in which the budget constraint binds in each state) or across all states (if the budget constraint binds in expectation). Update the consumption distribution in each state under the updated marginal tax rate schedule and state-specific income distribution.

6. Repeat steps 3–5 until the updating of the marginal tax rate schedule and the difference between the proposed alternative tax schedule and the current tax schedule are trivially small.

7. Check the second-order condition (SOC) that income is non-decreasing in income-earning ability in each state at the fixed point.

The resulting fixed-point marginal tax rate schedule satisfies the necessary FOC for robust optimality. This process does not guarantee convergence to a fixed-point tax schedule. However, if it does converge, and the SOC is satisfied, this represents the optimal tax schedule. This result depends on the fact that the individual utility functions satisfy the single-crossing property.-crossing property.

C.1.1. Step 1: Characterize Beliefs about the ETI in Different States

We characterize beliefs about the long-term ETI as a discrete probability distribution of states over the parameter σ. We generate a vector of elasticity states and a corresponding vector of probabilities assigned to each state s.[27] Within each state, we assume the value of the parameter σ_s is homogeneous across the population. In the specification based on beliefs elicited in our survey of academic economists, we assign a probability to each elasticity state equal to the unweighted average probability assigned to that state by the economists surveyed.[28] In specifications in which we assume certainty, a probability of 1 is assigned to a single elasticity state.

C.1.2. Step 2: Initialize Each State at the Status Quo

We begin by importing the discrete status quo US market income distribution $H_{T_{us}}(y)$, consumption distribution, and corresponding probability mass function $f(\theta)$ for types $\theta \in \Theta$ from Piketty et al. (2018). We adjust $H_{T_{us}}(y)$ by imposing a Pareto tail above the 90th percentile income, use linear interpolation to adjust consumption accordingly for income earners above that threshold, and generate a cumulative distribution function of types $F(\theta)$.[29] Next, we calibrate the current US tax system. We obtain the status quo marginal tax rate schedule $T'_{US}(y)$ by setting the lump-sum grant equal to the lowest income earner's consumption level and mapping the empirical income and consumption distributions to implied marginal tax rates. In the case in which the budget constraint binds in expectation, we initialize a single lump-sum grant, whereas we initialize a vector of state-specific lump-sum grants in the case in which the budget constraint binds in each state. The methodology used in Piketty et al. (2018) ensures that pretax and posttax national incomes are equal to reconcile with national accounts. As a result, under this calibration we have an exogenous revenue requirement of $R = 0$.[30]

For each elasticity state, we initialize an income and consumption distribution at the status quo. In addition, we set a single fixed grid of income levels along which the robust optimal marginal tax rate schedule $T^{*\prime}(y)$ will be defined. We initialize the robust marginal tax rate schedule via linear interpolation of $T'_{US}(y)$ to each income level in the fixed grid of incomes. We also initialize state-specific marginal tax rate schedules at $T'_{US}(y)$ to serve as dynamic mappings from the robust tax schedule defined along the fixed grid of incomes to the state-specific income distributions.

Finally, we use $H_{T_{us}}(y)$, $T'_{US}(y)$, and the parameter σ_s in each state to compute an underlying state-specific implied income-earning ability distribution using the FOC from the individual's utility maximization problem. We assume the individual utility function takes the Type 1 form from Saez (2001) with the logarithmic transformation

$$U(c_s^\theta, y_s^\theta) = \ln\left(c_s^\theta - \frac{1}{1 + (1/\sigma_s)} \left(\frac{y_s^\theta}{w_s^\theta} \right)^{1+(1/\sigma_s)} \right). \tag{A32}$$

For type θ in state s with status quo income y^{θ}_{US}, the implied ability is

$$w^{\theta}_{s} = \left(y^{\theta}_{US}(1 - T'_{US}(y^{\theta}_{US}))^{-\sigma_s}\right)^{\frac{1}{1+\sigma_s}}. \tag{A33}$$

We then solve for the labor disutility constants k^{θ}_{s} in each state that ensure that utility and consequently the marginal social welfare weight g^{θ}_{s} for each type is equal across all states in the status quo.[31] Furthermore, we impose the normalization that $k^{\theta}_{s} = 0$ under the central parameter value of $\sigma_{\text{base}} = 0.4$ from our survey of academic economists, which implies

$$k^{\theta}_{s} = \frac{1}{1 + (1/\sigma_s)}\left(\frac{y^{\theta}_{US}}{w^{\theta}_{s}}\right)^{1+(1/\sigma_s)} - \frac{1}{1 + (1/\sigma_{\text{base}})}\left(\frac{y^{\theta}_{US}}{w^{\theta}_{\text{base}}}\right)^{1+(1/\sigma_{\text{base}})}. \tag{A34}$$

C.1.3. Step 3: Update the Marginal Social Welfare Weights and Marginal Tax Rate Schedule

First, we compute the marginal social welfare weight for each type in each state as $g^{\theta}_{s} = \Phi'(U^{\theta}_{s})/\lambda$.[32] Here, $\Phi(x) = \ln(x)$ and λ is the marginal value of public funds, with $\lambda = E_s[\int \Phi'(U^{\theta}_{s})dF(\theta)]$.[33] When the budget constraint binds in each state and the social value of marginally increasing the lump-sum grant is not equal across all states, λ is the expected marginal value of public funds and we further define the state-specific marginal value of public funds: $\lambda_s = \int \Phi'(U^{\theta}_{s})dF(\theta)$.

We now employ the result, from equation (24) in the text, that when the budget constraint binds only in expectation, the robust optimal tax satisfies

$$\frac{T^{*\prime}(y)}{1 - T^{*\prime}(y)} = \frac{E_s\left[\int_y^{\infty}(1 - \hat{g}_s(z))\, h_{s,T}(z)dz\right]}{yE_s[\zeta_s(y)h_{s,T}(y)]}, \tag{A35}$$

with state-specific endogenous income densities $h_{s,T}(y)$, compensated ETI $\zeta_s(y)$, and marginal social welfare weights $\hat{g}_s(y)$. Note that in the absence of income effects, $\hat{g}_s(y) = g_s(y)$. To implement this calculation in the simulations, we use the fact that $E_s[\int_0^{\infty}(1 - g_s(z))h_{s,T}(z)dz] = 0$ and $\zeta_s(y) = -(\partial y/\partial(1 - T'(y)))_s \cdot ((1 - T'(y))/y)$ to rewrite the condition above as

$$T^{*\prime}(y) = \frac{E_s\left[\int_0^{y}(g_s(z) - 1)h_{s,T}(z)dz\right]}{E_s\left[\left(\frac{\partial y}{\partial(1 - T^{*\prime}(y))}\right)_s h_{s,T}(y)\right]}. \tag{A36}$$

The FOC from the individual of type θ's utility maximization problem in state s is $(1 - T'(y_s^\theta)) - (y_s^\theta/w_s^\theta)^{1/\sigma_s} \cdot (1/w_s^\theta) = 0$. Implicitly differentiating with respect to the marginal "keep rate" $1 - T'(y_s^\theta)$, we can solve for the local labor supply response:

$$\zeta_s(y) = \left(\frac{\partial y}{\partial(1 - T'(y))}\right)_s \cdot \frac{1 - T'(y)}{y} = \frac{\sigma_s}{1 + \frac{T''(y)}{1-T'(y)}\sigma_s y}. \tag{A37}$$

In the case where the budget constraint binds in each state, the optimal tax condition satisfies equation (29) from the text:

$$\frac{T^{*\prime}(y)}{1 - T^{*\prime}(y)} = \frac{E_s\left[\int_y^\infty (\lambda_s - \hat{g}_s(z)\lambda)h_{s,T}(z)dz\right]}{yE_s[\lambda_s\zeta_s(y)h_{s,T}(y)]}. \tag{A38}$$

In this case, we use the fact that $\int_0^\infty(\lambda_s - g_s(z)\lambda)h_{s,T}(z)dz = 0$ in each state to rewrite the above condition as

$$T^{*\prime}(y) = \frac{E_s\left[\int_0^y (g_s(z)\lambda - \lambda_s)h_{s,T}(z)dz\right]}{E_s\left[\lambda_s\left(\frac{\partial y}{\partial(1 - T^{*\prime}(y))}\right)_s h_{s,T}(y)\right]}. \tag{A39}$$

We compute the alternative tax schedule from the right-hand side of equation (A36) or (A39), depending on the specification, at each point in our fixed grid of income levels. We use the current tax schedule in place of $T^{*\prime}(y)$ and employ linear interpolation to map from inputs computed under state-specific income distributions to the fixed income grid. We update the marginal tax rate schedule as the weighted average of the alternative tax schedule and the current tax schedule. We apply a weight of 0.001 to the alternative tax schedule to aid in a gradual progression toward convergence to the fixed-point tax schedule. We also impose an upper bound of 1.0 and a lower bound of –0.1 on the alternative tax schedule to facilitate convergence; these limits do not bind at the optimum in any of our specifications.

C.1.4. Step 4: Determine the Optimal Labor Supply for Each Type in Each State

We rearrange the FOC from the individual's utility maximization problem, yielding

$$y_s^\theta = w_s^{\theta(1+\sigma_s)} \cdot (1 - T'(y_s^\theta))^{\sigma_s}. \tag{A40}$$

Due to the endogeneity of the marginal tax rate, we employ another fixed-point algorithm to reoptimize the labor supply choices made by each type. It proceeds as follows:

1. Use linear interpolation to map from the marginal tax rate schedule on our fixed grid of incomes to the current income distribution in each state.[34]
2. Compute an alternative income distribution from the right-hand side of equation (A40), using the marginal tax rate corresponding to the current income of each type in each state.
3. Update the income distribution in each state as the weighted average of the alternative income distribution and the current income distribution. Apply a weight of 0.05 to the alternative income distribution to aid in a gradual progression toward convergence to the fixed-point income distribution.[35]
4. Repeats steps 1–3 until the updating of the income distribution is trivially small.[36]

The resulting income distribution represents the optimal labor supply choices under the updated marginal tax rate schedule.

C.1.5. Step 5: Update the Lump-Sum Grant and Consumption in Each State

After updating the marginal tax rate schedule and arriving at the corresponding fixed-point income distribution, we update the lump-sum grant(s) and the state-specific consumption distributions. First, we simply update the lump-sum grant to ensure that the budget constraint is either satisfied in expectation (i.e., $E_s[\int T(z)h_{s,T}(z)dz] = R$) or in each state (i.e., $\int \mathcal{T}_s(z)h_{s,T}(z)dz = R$), depending on the specification. Second, we update consumption in each state using the updated income distribution, lump-sum grant, and marginal tax rate schedule.

C.1.6. Step 6: Converge to the Fixed-Point Tax Schedule

We repeat steps 3–5 until (i) the updating of the marginal tax rate schedule is trivially small and (ii) the difference between the proposed alternative tax schedule and the current tax schedule is trivially small. For criterion (i), we define a trivially small update as occurring when the magnitude of the vector of differences between the current and updated tax schedules is less than 10^{-5}. For criterion (ii), we define a trivially small difference as present when the percentage of difference in marginal tax rates at all income levels in our fixed grid is less than 0.01%.[37] Once these criteria are satisfied, the procedure is complete and the resulting tax schedule is the fixed-point tax schedule.[38]

C.1.7. Step 7: Check for Optimality of the Fixed-Point Tax Schedule

Our final step is to confirm that the fixed-point tax schedule is the robust optimal nonlinear tax schedule. We check the SOC that the fixed-point incomes y_s^θ corresponding to the fixed-point tax schedule are non-decreasing in income-earning ability type in each state.

C.1.8. Impose an Asymptotic Marginal Tax Rate at High Incomes

Because the income-earning ability distribution is bounded in each state, Seade's (1977) "zero at the top" result arises and the marginal tax rate at top incomes declines toward zero. Realistically, the policymaker may not know the top income-earning ability with certainty in each state, so we impose the marginal tax rate that plateaus above $300,000 in income across all incomes above that threshold. We then employ the labor supply fixed-point algorithm from step 4 to reoptimize labor supply choices under the modified tax policy. Finally, we update the lump-sum grant(s) and consumption distributions as in step 5.

C.1.9. Enforce Within-State Budget Constraints

For comparison to the robust optimal tax policy under within-state budget constraints, we modify the fully prespecified best estimate tax policy—optimized under a certain, best estimate value of σ—to enforce within-state budget constraints. The modification proceeds as follows:

1. Employ the labor supply fixed-point algorithm in step 4 to optimize the labor supply of each type in each elasticity state under the marginal tax rate schedule of the fully prespecified best estimate tax policy and the underlying state-specific income-earning ability distributions implied by each value of σ.
2. Compute state-specific lump-sum grants, ensuring that the budget constraint is satisfied in each state under the best estimate policy's marginal tax rate schedule and the updated income distributions from the previous step. Compute the consumption of each type in each state as

$c_s^\theta = y_s^\theta - T(y_s^\theta) + b_s$ using the best estimate policy's marginal tax rate schedule, the updated income distributions, and the state-specific lump-sum grants.

C.1.10. Modifications for a Simple Linear Tax Illustration

We modify steps 2 and 3 of the fixed-point algorithm detailed above as follows to implement a simple linear tax illustration:

Step 2: We define an economy with two income-earning ability types. Each type constitutes 50% of the population. In the status quo, the low-ability type earns the 25th percentile income from the status quo US income distribution, whereas the high-ability type earns the 75th percentile income. Each type's consumption is equal to the empirical consumption that corresponds to its income from the status quo US consumption distribution, adjusted by the same lump sum. The lump-sum adjustment is set to ensure the budget is balanced in the status quo and the exogenous revenue requirement R is zero given a linear tax system.

Step 3: We instead employ the result from the proof of proposition 1 in the text that when the budget constraint binds in expectation, the robust optimal tax satisfies

$$\frac{t^*}{1-t^*} = \frac{\sum_s \pi_s \bar{y}_s - \sum_s \pi_s \left[\sum_\theta p_s^\theta g_s^\theta y_s^\theta\right]}{\sum_s \pi_s [\zeta_s \bar{y}_s]}, \tag{A41}$$

where, in our simple economy, $\bar{y}_s$ is the average income in each state across the two types and $p_s^\theta = 0.5$ for all θ. To implement the computation of the alternative linear tax rate, we rearrange and define t_0^* as the current linear tax rate, which in the notation of this section yields

$$t^* = \frac{E_s[\bar{y}_s] - E_s\left[\sum_\theta p_s^\theta g_s^\theta y_s^\theta\right]}{E_s[\zeta_s \bar{y}_s]} \cdot (1 - t_0^*). \tag{A42}$$

We compute the alternative tax rate from the right-hand side. Similarly, when the budget constraint binds in each state, we employ the result from the proof of proposition 1 in the text that the robust optimal tax satisfies

$$\frac{t^*}{1-t^*} = \frac{\sum_s \pi_s \lambda_s \bar{y}_s - \lambda \sum_s \pi_s \left[\sum_\theta p_s^\theta g_s^\theta y_s^\theta\right]}{\sum_s \pi_s [\lambda_s \zeta_s \bar{y}_s]}. \tag{A43}$$

Again, we rearrange and rewrite, yielding

$$t^* = \frac{E_s[\lambda_s \bar{y}_s] - \lambda E_s\left[\sum_\theta p_s^\theta g_s^\theta y_s^\theta\right]}{E_s[\lambda_s \zeta_s \bar{y}_s]} \cdot (1 - t_0^*), \tag{A44}$$

and we compute the alternative tax rate from the right-hand side. The rest of the procedure remains the same as in the nonlinear cases.

C.2. Details of the Welfare Computations

We estimate the welfare effects of switching from the best estimate tax policy to the robust optimal tax policy under the same beliefs about the probability distribution of σ in the following steps:

1. Use the utility function in equation (A32) and the income-earning ability, income, and consumption distributions in each elasticity state at the optimum to compute the utility of each type in each state under the robust optimal tax policy.
2. a) In the case of fully prespecified policies: Employ the labor supply fixed-point algorithm in step 4 of Section C.1 to optimize the labor supply of each type in each elasticity state under the marginal tax rate schedule of the best estimate tax policy. Compute the consumption of each type in each state using the optimal labor supply in each state and the prespecified grant and marginal tax rate schedule of the best estimate tax policy. Use the individual's utility function and the income-earning ability, income, and consumption distributions in each state to compute the utility of each type in each state.
   b) In the case of within-state budget constraints: Repeat step 1 for the best estimate tax policy.
3. Compute the change in utility for each type in each state from switching from the best estimate tax policy to the robust optimal tax policy.
4. Compute the marginal value of public funds λ_s under the best estimate tax policy in each state as $\lambda_s = \int \Phi'(U_s^\theta)dF(\theta)$, where $\Phi(x) = \ln(x)$. Convert the change in utility in each state from utils to dollars by dividing by λ_s.
5. Aggregate the money-metric change in utility across the population using the probability mass function of types $f(\theta)$ and an estimate of the US population. Divide by aggregate consumption in each state under

the best estimate tax policy to rescale the welfare effect as a percentage of consumption.

C.3. Implementation in MATLAB

We use the five MATLAB files detailed below to implement the fixed-point algorithm:

- "run_simulations.m" executes the simulations.
- "economy.m" defines an `economy`-class object that implements the fixed-point algorithm for the case in which the budget constraint binds in expectation.
- "economy_budgetbind.m" defines a subclass that modifies the `economy` class to enforce within-state budget constraints.
- "economy_linear.m" defines a subclass that modifies the `economy` class to implement the fixed-point algorithm in a simple setting with two income-earning ability types, a linear tax system, and a budget constraint that binds in expectation.
- "economy_linear_budgetbind.m" defines a subclass that modifies the `economy_linear` subclass to enforce within-state budget constraints.

We also use the two MATLAB files below to modify `economy`-class objects generated under a certain, best estimate value of σ to enforce within-state budget constraints in each state of the probability distribution of σ.

- "balance_within_state.m" modifies `economy`-class and `economy_budgetbind`-subclass objects.
- "balance_within_state_linear.m" modifies `economy_linear`-subclass and `economy_linear_budgetbind`-subclass objects.

We use one MATLAB file to impose an asymptotic marginal tax rate at high incomes:

- "impose_top_rate.m" modifies `economy`-class and `economy_budgetbind`-subclass objects.

We use an additional MATLAB file to compute the welfare effects of changing tax policies:

• "compute_welfare_effect.m" defines a function that computes the money-metric welfare effect of switching from the best estimate tax policy to the robust optimal tax policy optimized under the same beliefs about the probability distribution of σ.

All of the files above can be found in the "code" directory of this paper's replication files. To generate the results reported in this paper, run the script "run_simulations.m" in MATLAB. It will output "numbersfortext.tex" directly to the "output" folder. The file "numbersfortext.tex" contains LaTeX commands with the survey probabilities, lump-sum grants, and money-metric welfare effects reported in this paper as well as the contents of table 1. The script will also output all PDF figures to the "Figures" subfolder of the "output" folder.

Endnotes

Author email addresses: Lockwood (ben.lockwood@wharton.upenn.edu), Weinzierl (mweinzierl@hbs.edu). University of Pennsylvania Wharton School and NBER; University of Pennsylvania Wharton School; Harvard Business School and NBER. We thank the editor and participants at the NBER Tax Policy and the Economy 2020 conference for valuable comments and suggestions. We thank the Wharton Dean's Research Fund for financial support. For acknowledgments, sources of research support, and disclosure of the authors' material financial relationships, if any, please see https://www.nber.org/books-and-chapters/tax-policy-and-economy-volume-35/designing-not-checking-policy-robustness-example-optimal-taxation.

1. We will use the term "uncertainty" as shorthand for risk, uncertainty, and ignorance, three classes of limits to understanding (see Zeckhauser 2014). Our analysis has the policy designer face risk (known probabilities of known states) and uncertainty (unknown probabilities of known states), and we use the latter term to encompass both. To be specific, we assume known probabilities of known states or have the policymaker attempt to resolve uncertainty over the unknown probabilities of known states (i.e., via a survey of economists). We would be eager to incorporate ignorance (unknown probabilities of unknown states), but doing so would take us too far from standard policy evaluation methods. See Weinzierl (2017) for a possible approach to managing ignorance in policy design.

2. We address primarily economists working in the welfarist tradition widely assumed in modern economic analysis. A concern for robustness may lie behind support for nonwelfarist principles of policy design, though we do not explore that possibility here. See Weinzierl (2017, 2018) for an elaboration of the case for nonwelfarist principles.

3. Although we have not seen our proposed approach to robustness applied in tax policy design, there is some precedent for this approach in environmental policy: "The present study takes the standard economic approach to uncertainty known as the expected utility model, which relies on an assessment with subjective or judgmental probabilities. This approach uses the best available estimates of the averages and uncertainties for the major variables to determine how the presence of uncertainty might change our policies relative to a best-guess policy" (Nordhaus 2007, 33).

4. In doing so, we aggregate many individuals' beliefs, and for simplicity we apply equal weights to each. If some sources of beliefs are more credible than others, their influence would be greater on the policy designer's probability distribution over states.

5. As Saez et al. (2012, 13) write in their literature review on this elasticity, "The long-term response is of most interest for policy making."

6. Note that we designate the ETI as containing within it all responses to tax policy that affect income, e.g., including human capital accumulation decisions. Changes to the income distribution not captured by this elasticity may be due to nontax policy changes (such as education) and changes to the productive environment (such as technology). As there is no other income in the model, and the model is static (it can be thought of as a lifetime model), there is no income shifting in response to a tax change.

7. As Saez et al. (2012, 13, 14) point out, "the long-term response is more difficult to identify empirically. The empirical literature has primarily focused on short-term (one year) and medium-term (up to five year) responses, and is not able to convincingly identify very long-term responses." In part, they argue, this difficulty is because the ETI is not a "structural parameter" with a value independent of context. Instead, it depends on the existing tax and broader economic system, such that "an elasticity estimated around the current tax system may not apply to a hypothetical large tax change." Factors of potential importance on which empirical evidence is quite limited, and which thereby complicate the econometrics, include the long-term responsiveness of investment in human capital and interdependent social norms.

8. This uncertainty is closely related to the assumed unobservability of income-earning ability at the heart of the Mirrlees model. Both the income and ability distributions depend, in the long term, on the evolution of a range of unforeseeable factors such as production technology, the educational system, fertility choices, and various nontax policies.

9. The inverse-optimum literature (see Bourguignon and Spadaro 2012; Bargain et al. 2014; Lockwood and Weinzierl 2016) seeks to infer welfare weights from existing policy choices. Its results are not easily summarized, but some analyses have found implied welfare weights that do not decline monotonically or as quickly as the standard utilitarian approach would imply. See also Weinzierl (2014), Saez and Stantcheva (2016), and references therein.

10. At the optimum (and absent income effects), the policymaker must be indifferent between a marginal dollar of public funds and a marginal increase in the lump-sum grant, implying that $\lambda = \Sigma_j(dV^j/db)$, and in turn implying that marginal social welfare weights average to one.

11. In Section A, we also derive the optimal linear tax rates using direct constrained optimization, resulting in the same solution.

12. Here we interpret π_s as the probabilities of each state as perceived by the policymaker, whether arising from a known probability distribution or (more likely) subjective uncertainty based on the policymaker's assessment of empirical evidence or aggregation of divergent views from experts.

13. Importantly, although this setup implies some transfer of resources across states, it does not allow the tax authority to choose state-specific lump-sum grants optimally, e.g., by increasing the grant in states where labor disutility is high on average.

14. The contrast between our approach and that of Hansen and Sargent is analogous to the contrast, pointed out by Arrow (1973), between economists' conventional utilitarianism and the influential Difference Principle of Rawls (1971). Rawls reasoned that individuals designing society's institutions from behind a veil of ignorance, and thereby ignorant of their own positions within that society, would maximize the well-being of the worst-off individual in that society. Arrow argued against this inference as reflecting too extreme a level of risk aversion, in particular relative to Harsanyi's (1953, 1955) inference (using a similar thought experiment designed to emphasize impartiality) that social welfare ought to be the probability-weighted sum of individual utilities within society. Thus our model takes Arrow's view with respect to uncertainty over policy parameters. In principle, our approach can be extended to incorporate risk aversion across states on the part of the policymaker by replacing the objective in equation (7) with $\Sigma_s \pi_s \Phi(\Sigma_i p_s^i V_s^i(a, b^*))$, where Φ is a concave transformation; then the Hansen and Sargent (2001) notion of robustness corresponds to the limiting case of extreme concavity in Φ.

15. If uncertainty is asymmetric in other ways—for example, if the distribution of elasticities has a mean greater than its median—this difference between the robust optimal policy and the best estimate policy is potentially even greater. And a similar logic applies to the terms in the numerator: to the extent that variation across states in parameters generates asymmetric uncertainty in the welfare effects of taxation, the robust optimal policy will differ from the best estimate policy.

16. Because this model takes a broad perspective of the tax and spending system as an integrated whole, this can be loosely interpreted as government spending on social support and public services adjusting to balance the budget rather than a literal cash grant paid out to households.

17. In the case of quasilinear utility, which we assume in the linear tax case earlier but relax here, income effects are absent and $\hat{g}(y)$ reduces to $g(y)$.

18. See note 1 for a discussion of these three terms.

19. As do we, these authors note that the uncertainty surrounding the parameters on which their work focuses extends to many areas of economics, writing: "Many of these problems are not unique to identifying the long-run ETI, but apply to the estimation of all behavioral responses" (43).

20. Real responses are those that change income earned, not reported, and are the relevant ones for optimal tax policy (because enforcement is a concern outside the theory).

21. These values are imputed by this paper's authors, using Jones's optimal top marginal tax rates from his table 1 and the standard (see Diamond and Saez 2011) formula for the top marginal tax rate with a Pareto parameter $\alpha = 1.5$.

22. We are assuming that income is the product of these abilities and labor supply as determined by the individual's optimization.

23. When the tax function is locally curved (i.e., $T'' \neq 0$), the ETI that is relevant for tax policy differs from the structural parameter. Intuitively, a change in the marginal tax rate induces an adjustment in y, which further changes the marginal tax rate due to the curved tax function, inducing a further adjustment in y, and so on. The ETI ζ is defined to include the full effect of this iteration. See equation (A37) in Section C for the precise relationship between ζ and σ.

24. Many papers have studied the implications of behavioral frictions for optimal income taxation. See Rees-Jones and Taubinsky (2018) for a broad discussion. For examples of specific applications, see Allcott, Lockwood, and Taubinsky (2018) for a model with less than fully salient income taxes and Lockwood (2020) for a model with present-biased workers.

25. We note that the key driver of these comparisons is that the optimal tax rate in every case is substantially higher in the status quo, implying that a higher ETI results in lower total income. If instead the policy were considered from the perspective of a status quo with higher than optimal tax rates, the high ETI states would instead generate more income.

26. For comparison, we also compute these expected welfare gains under the illustrative distributions used for the simulations in figures 3 and 4. The welfare effect increases in the skewness of the distribution of σ because the robust policy is relatively more efficient at targeting tax rates across incomes when the discrepancy between the elasticities in each state and the best estimate of the elasticity is greater. In the setting of figure 4, the welfare gains are the largest, estimated at 4.0% and 1.3% of consumption, or $724 billion and $288 billion, when the budget constraint binds in expectation and in each state, respectively. Under less skewness in the distribution of σ as in figure 3, the estimated welfare gains are 2.1% and 0.4% of consumption, or $380 billion and $127 billion, respectively.

27. We define $E_s[x_s] \coloneqq \Sigma_s \pi_s x_s$, where π_s is the probability assigned to state s.

28. We map from the intervals over which we elicited beliefs in the survey to specific values and assign the average probabilities to those values.

29. The Pareto parameter is chosen such that the local parameter just below the 90th percentile income is equal to the constant parameter above the 90th percentile income. We also modify $f(\theta)$ along the Pareto tail by defining additional types at low and middle incomes and fewer types at high incomes to facilitate convergence to the fixed-point tax schedule.

30. To enforce this equality in the presence of our numerical integration, which generates approximations, we rescale status quo consumption by a constant factor (about 1.01) so national income and consumption are exactly equal in the status quo. This does not preclude deficit spending; rather, it requires deficits to be accounted for in income. Piketty et al. (2018, 573) include deficit-funded transfers in consumption and adjust incomes by "allocat[ing] 50% of the deficit proportionally to taxes paid, and 50% proportionally to government spending received."

31. In the status quo, these state-specific constants are necessary to ensure that each individual's status quo labor supply choice maximizes each individual's utility, regardless of which elasticity state happens to obtain.

32. We verify that $E_s[\int g_s^\theta dF(\theta)] \approx 1$.
33. Our assumption that there are no income effects simplifies the computation of the marginal value of public funds and marginal social welfare weights. All integrals are computed numerically by the trapezoidal method, using the *trapz* function in MATLAB.
34. We limit the marginal tax rates to an upper bound of 0.99 to avoid nonreal values; this limit does not bind at the optimum in any of our specifications. Furthermore, we enforce the "zero at the top" result that arises from using a bounded ability distribution (see Seade 1977) when the marginal tax rate of the top ability type is extrapolated to be negative.
35. To facilitate convergence, we impose a bounding procedure that ensures incomes do not become "too large" during the iteration process, while still remaining monotonic. Specifically, we limit the values of the new income distribution so that given proposed income $y_{s_0}^\theta$ from step 2, we compute $y_s^\theta = \min\{y_{s_0}^\theta, 10^9 \cdot 10^{(n-1)/(N-1)}\}$, where n is type θ's rank (of N) in the income distribution.
36. We define a trivially small update as occurring when the maximum of the absolute value of the pairwise differences between the incomes under the current and updated income distributions is less than 0.0001.
37. We exclude the tax rate for the top income earners, which is expected to be zero, when checking this criterion to avoid division by zero.
38. Due to the "zero at the bottom" result that arises from using a bounded ability distribution (see Seade 1977), we do not plot the tax rate for the lowest income in our fixed grid of incomes because rates will approach zero at any arbitrary first income cell.

References

Allcott, Hunt, Benjamin Lockwood, and Dmitry Taubinsky. 2018. "Ramsey Strikes Back: Optimal Commodity Tax and Redistribution in the Presence of Salience Effects." *AEA Papers and Proceedings* 108:88–92.

Arrow, Kenneth J. 1973. "Rawls's Principle of Just Saving." *Swedish Journal of Economics* 75 (4): 323–35.

Bargain, Olivier, Mathias Dolls, Dirk Neumann, Andreas Peichl, and Sebastian Siegloch. 2014. "Tax-Benefit Revealed Social Preferences in Europe and the US." *Annals of Economics and Statistics* (113/114):257–89.

Bourguignon, François, and Amedeo Spadaro. 2012. "Tax–Benefit Revealed Social Preferences." *Journal of Economic Inequality* 10 (1): 75–108.

Chetty, Raj. 2012. "Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply." *Econometrica* 80 (3): 969–1018.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review* 101 (3): 471–75.

Diamond, Peter A. 1975. "A Many-Person Ramsey Tax Rule." *Journal of Public Economics* 4 (4): 335–42.

———. 1998. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *American Economic Review* 88 (1): 83–95.

Diamond, Peter, and Emmanuel Saez. 2011. "The Case for a Progressive Tax: From Basic Research to Policy Recommendations." *Journal of Economic Perspectives* 25 (4): 165–90.

Giertz, Seth. 2009. "The Elasticity of Taxable Income: Influences on Economic Efficiency and Tax Revenues, and Implications for Tax Policy." In *Tax Policy Lessons from the 2000s*, ed. Alan Viard, 101–36. Washington, DC: AEI Press.

———. 2010. "The Elasticity of Taxable Income during the 1990s: New Estimates and Sensitivity Analyses." *Southern Economic Journal* 77 (2): 406–33.

Hansen, Lars Peter, and Thomas J. Sargent. 2001. "Acknowledging Misspecification in Macroeconomic Theory." *Review of Economic Dynamics* 4 (3): 519–35.
Harsanyi, John C. 1953. "Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking." *Journal of Political Economy* 61 (5): 434–35.
———. 1955. "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility." *Journal of Political Economy* 63 (4): 309–21.
Jones, Charles I. 2019. "Taxing Top Incomes in a World of Ideas." Working Paper no. 25725, NBER, Cambridge, MA.
Kasy, Maximilian. 2018. "Optimal Taxation and Insurance Using Machine Learning—Sufficient Statistics and Beyond." *Journal of Public Economics* 167:205–19.
Keane, Michael, and Richard Rogerson. 2012. "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." *Journal of Economic Literature* 50 (2): 464–76.
Lockwood, Benjamin B. 2020. "Optimal Income Taxation with Present Bias." *American Economic Journal Economic Policy* 12 (4): 298–327.
Lockwood, Benjamin B., and Matthew Weinzierl. 2016. "Positive and Normative Judgments Implicit in US Tax Policy, and the Costs of Unequal Growth and Recessions." *Journal of Monetary Economics* 77:30–47.
Manski, Charles F. 2009. "Social Planning with Partial Knowledge of Social Interactions." *Journal of Institutional and Theoretical Economics (JITE)/Zeitschrift für die gesamte Staatswissenschaft* 165:146–58.
Mirrlees, James A. 1971. "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies* 38 (2): 175–208.
Nordhaus, William. 2007. "The Challenge of Global Warming: Economic Models and Environmental Policy." http://www.econ.yale.edu/~nordhaus/homepage/OldWebFiles/DICEGAMS/dice_mss_072407_all.pdf.
Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman. 2018. "Distributional National Accounts: Methods and Estimates for the United States." *Quarterly Journal of Economics* 133 (2): 553–609.
Rawls, John. 1971. *A Theory of Justice*. Cambridge, MA: Belknap Press of Harvard University Press.
Rees-Jones, Alex, and Dmitry Taubinsky. 2018. "Taxing Humans: Pitfalls of the Mechanism Design Approach and Potential Resolutions." *Tax Policy and the Economy* 32 (1): 107–33.
Saez, Emmanuel. 2001. "Using Elasticities to Derive Optimal Income Tax Rates." *Review of Economic Studies* 68 (1): 205–29.
Saez, Emmanuel, Joel Slemrod, and Seth H. Giertz. 2012. "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review." *Journal of Economic Literature* 50 (1): 3–50.
Saez, Emmanuel, and Stefanie Stantcheva. 2016. "Generalized Social Marginal Welfare Weights for Optimal Tax Theory." *American Economic Review* 106 (1): 24–45.
Seade, Jesus K. 1977. "On the Shape of Optimal Tax Schedules." *Journal of Public Economics* 7 (2): 203–35.
Sheshinski, Eytan. 1972. "The Optimal Linear Income-Tax." *Review of Economic Studies* 39 (3): 297–302.
Weinzierl, Matthew. 2014. "The Promise of Positive Optimal Taxation: Normative Diversity and a Role for Equal Sacrifice." *Journal of Public Economics* 118:128–42.
———. 2017. "A Welfarist Role for Nonwelfarist Rules: An Example with Envy." Working Paper no. 23587, NBER, Cambridge, MA.

———. 2018. “Revisiting the Classical View of Benefit-based Taxation.” *Economic Journal* 128 (612): F37–F64.

Williams, Noah. 2008. “Robust Control.” In *New Palgrave Dictionary of Economics*, ed. Steven Durlauf and Lawrence Blume, 2nd ed. London: Palgrave Macmillan.

Zeckhauser, Richard. 2014. “New Frontiers beyond Risk and Uncertainty: Ignorance, Group Decision, and Unanticipated Themes.” In *Handbook of the Economics of Risk and Uncertainty*, ed. Mark J. Machina and W. Kip Viscusi. Oxford: Elsevier.